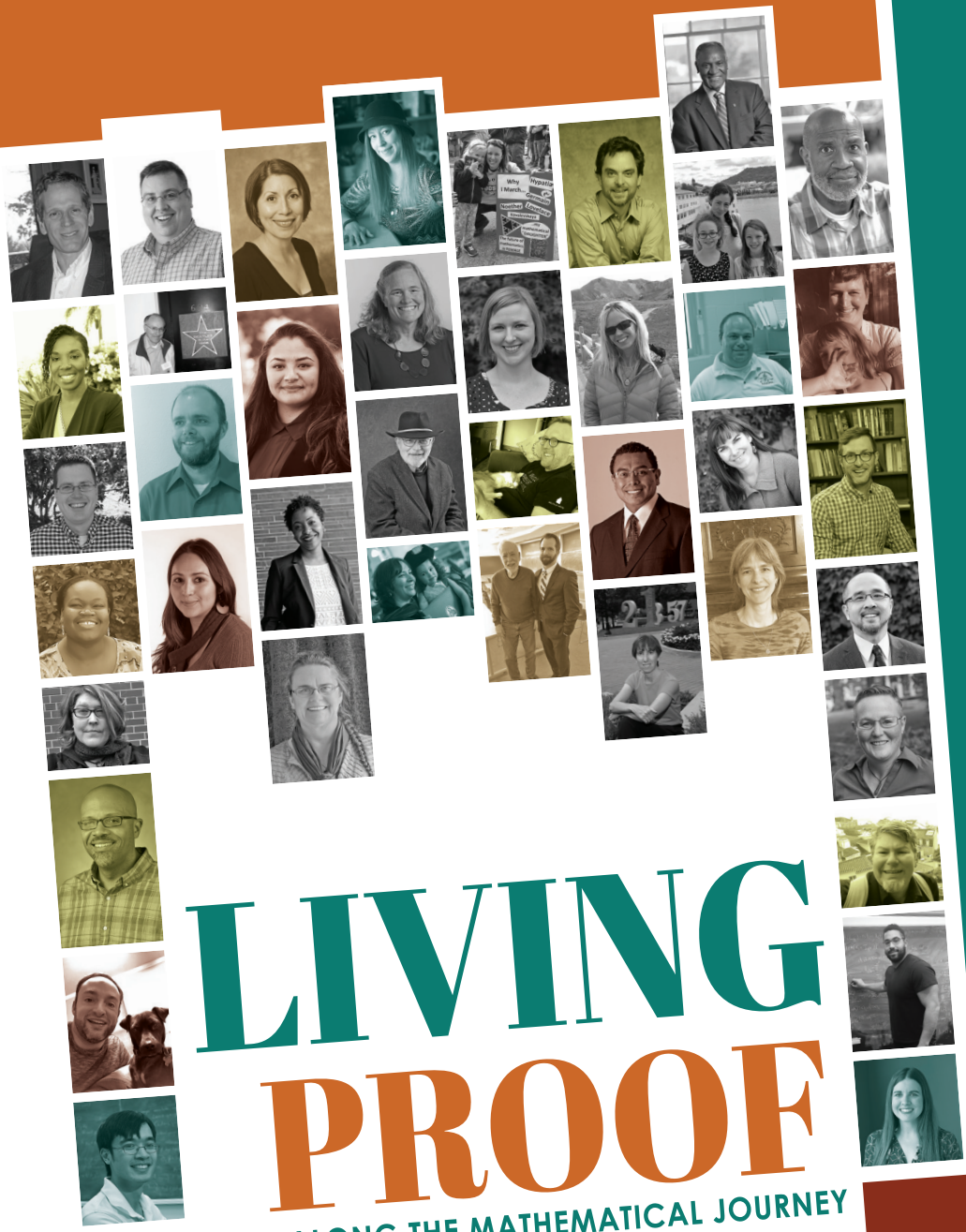

LIVING PROOF

STORIES OF RESILIENCE ALONG THE MATHEMATICAL JOURNEY

EDITED BY: ALLISON K. HENRICH
EMILLE D. LAWRENCE
MATTHEW A. PONS
DAVID G. TAYLOR

AMS AMERICAN MATHEMATICAL SOCIETY

MAA MATHEMATICAL ASSOCIATION OF AMERICA

Living Proof

Stories of Resilience Along the Mathematical Journey

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Living Proof

Stories of Resilience Along the Mathematical Journey

Edited by

Allison K. Henrich
Seattle University

Emille D. Lawrence
University of San Francisco

Matthew A. Pons
North Central College

David G. Taylor
Roanoke College

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For simply picking up this book and allowing us and our wonderful contributors to help you on your path to becoming a mathematician, this book is dedicated to you, the reader.

Foreword

"Math is hard," Barbie famously declared. Well, Barbie was right, but math is not uniquely hard. Playing the violin is hard, hitting a baseball is hard, and learning a second language is hard. What seems to make mathematics different from playing the violin or learning Chinese is that the struggle to play violin doesn't make people feel defeated and dumb. Somehow, when we encounter difficulties in mathematics, our natural tendency is to retreat, to think it's too hard, we're not smart enough, or we're not "math people." We allow ourselves to be defeated by the difficulty. We understand that learning to play the violin requires making many, many hours of horrible screeching sounds, that learning to speak Chinese means making error after error and not being understood. But, somehow, when it comes to mathematics, we fear making mistakes. We imagine that there are "math people" to whom it is all transparent and, if it doesn't come to us immediately, we must not be one of them.

There are no such people. People who succeed in mathematics, like people who learn a musical instrument or a new language, spend a lot of time not understanding and feeling frustration. The path to understanding in mathematics necessarily involves, in the words of Steve Klee (4), being "willing to struggle." It is strange that people do not understand this about mathematics when it is commonplace in essentially every other field of human endeavor. The people whose stories are in this book clearly understand this fact. Some of them, for example Lola Thompson (7) and Laura Taalman (8), avidly embrace the struggle, they seek out the experience of frustration and confusion because they have realized that persistence in the face of difficulty leads to the rewards of learning and growth.

There is something that, I think, is slightly different about the learning of mathematics. After the long, hard struggle for understanding what often happens is a crystalline perfection of understanding. The resulting knowledge is so obvious, so natural, that we don't understand our previous diffi-

culty. Matthew Pons (5) and Lloyd Douglas (2) tell stories in the first part of this volume that wonderfully illustrate this point. This retrospective clarity is part of the charm and beauty of mathematics, in hindsight it was all so inevitable—it could not have been any other way.

There are dangerous myths in mathematics. One of them is that there exist "math people," people to whom it all comes easily and is obvious. People who study the theory of learning are discovering that grit and persistence in the face of difficulty are much more important than any inherent talent in learning mathematics. Simply believing that study and struggle are more important to learning than innate ability leads (through productive study and struggle) to more learning and more understanding. There are no "math people," mathematical thinking is a fundamental part of every human's intellectual capacity. The people we label "good at math" are simply those who have taken the time and trouble to engage the struggle more deeply than others.

The myth of "talent" has pernicious consequences. Math teachers look for "talent" and encourage it. Unfortunately, and this is not a phenomenon unique to mathematics, we tend to see that "talent" in people who look like us. This has the effect of erecting barriers to entrance into mathematics among populations of people who don't look like the majority of mathematicians: women, people of color, people with disabilities. No one wants these barriers to be in place. But, they exist, and they make it difficult for many people to access mathematics. Teachers of mathematics can also, often unintentionally, wreak profound damage on students by explicitly, or implicitly, conveying an expectation of non-success. The flip side of this is that teachers can also, again often unintentionally, have a profound positive impact with a simple, kindly encouraging remark.

In the second part you can read many stories of discouragement, people who perceived that they were not welcome in mathematics either because of direct impediment by someone in authority, or simply cultural structures within the mathematical community that made the individual feel uninvited. What's notable here is that these writers persisted. One cannot but wonder, along with Robin Wilson (14), about the hordes of people who, perceiving hostility, must have prematurely abandoned mathematics. Wilson points out, "Belonging is a fundamental human need." The mathematical community has neglected that need too often, for too long, for too many people. As a teacher (and lover) of mathematics it is profoundly disturbing to realize how many of the folks in Part II had to motivate themselves by a desire to prove us—the teachers and gatekeepers of mathematics—wrong. Alice Silverberg (20), Arlie Petters (18), Candice Price (19), Angie Hodge

(15): mathematics is much richer because they persisted, but we never, or only much too late, told them that. There are also stories in Part II of folks who, in the words of Jennifer Bowen (16) quoting Mr. Rogers, "looked for helpers." These stories illustrate the power of mentoring, and simple human compassion. There are profound, and simple, lessons for mathematics teachers in this chapter. We should learn them.

Part III tells some stories of unconventional strategies for achieving mathematical success. Robert Allen (24) recommends good hygiene, David Neel (26) takes hikes in inappropriate footwear. Robin Blankenship (25) tells a darkly comic, but ultimately disturbing, story of the psychological manipulation she had to perform to pass algebraic topology. Amanda Ruiz (27) and Nick Scoville (28), among other lessons, give positive proof that the math subject GRE has a pernicious exclusionary power in mathematics. And Hortensia Soto (29) tells an affecting story of the powerful impact a compassionate teacher can have.

Any reasonably self-aware successful person is completely cognizant of the effect of serendipity and luck in their achievements. The mathematicians whose stories are related in Part IV all perfectly convey this truth. Most didn't really know where they were going, or how to get there. But, all of them kept working, kept learning, persisted (there's that word again), and all have had enormous success and, more importantly, rich and interesting lives and careers. There's an old Latin proverb usually translated as "Fortune favors the bold." It's not perfectly accurate, as the stories in this chapter demonstrate, Fortune actually favors the stubborn, the persistent. If you keep working, if you keep seeking, you'll be rewarded by serendipity, which is really just, as these stories remind, the habit of mind to be engaged and to notice when something good has happened.

The stories of struggle in this volume were collected in order to inspire mathematics students. And, they will serve that purpose. But they should also inspire mathematicians and educators. We can create a mathematical world where demoralizing, punishing struggle is not necessary. It will always be necessary for people to struggle within their own minds to master mathematics, but we need to teach our students to see, à la Laura Taalman, the power and glory of mathematical struggle. Using reason and logic and one's own mind one can achieve mathematical insight—this is the most awesome of our intellectual capabilities, it's part of the essence of being human. Somehow, as the stories in this volume make clear, we are failing our students: they are feeling beaten up and defeated by this struggle. The struggle *is* real; it need not be damaging and hurtful.

If you are a mathematics student reading this book, my hope for you is that you find yourself somewhere in these pages and you are inspired to persist. If you are a mathematics teacher, I hope you find in these pages the inspiration to relieve the pressure of demoralizing struggle from a student.

Stephen Kennedy
Carleton College

P.S. Thanks to the generosity and public-spirited cooperation of the AMS and the MAA, pdf copies of this book are available for free download from the websites of both societies. Visit ams.org or maa.org and search for Living Proof.

Preface

This project grew out of conversations with students about the difficulties inherent in the study of mathematics. Many undergraduates have not yet learned to embrace the ups and downs that each of us faces as we make our way through the discipline, though many students make light of the challenges. We frequently hear statements like "the struggle is real" or "I'm on the struggle bus today" when they come with questions. Beneath these flippant phrases, there are insecurities about their own abilities, uncertainty about whether they have made the correct choice for a major, and a myriad of other emotions. And these are just the things that rise to the surface. For many students, there are also stereotypes and identity issues that influence their attitudes toward the discipline. Math should be difficult, as should any worthwhile endeavor. But it should not be crippling. The ability to succeed in a mathematical program should not be hindered by a person's gender, race, sexuality, upbringing, culture, socio-economic status, educational background, or any other attribute. Our primary goal in collecting this volume of essays is to push the conversation forward. Yes, math is difficult. We should talk about what makes it difficult. But we should also acknowledge the various biases and prejudices that people bring to their study of math that compound its difficulty. By making an effort to understand what we have in common and what makes our experiences different, our hope is that our community will become more inclusive while making the struggle more bearable—perhaps even more fun.

As you read this, we hope that you will find some inspiration and common ground in these pages. We trust that there is at least one story here that you can connect with. For those stories that you cannot relate to, we hope that you will come to better appreciate the diversity of our mathematical community and the challenges that others have faced. We also hope that you will laugh with some of our authors as they recount some of the more absurd struggles they have faced. In the end, we hope that you are motivated

to share your own stories as you learn more about the experiences of the people in your own mathematical lives.

Emille, Allison, Matthew, and David

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I

Mathematics Just Suddenly Feels Hard!

Many students who find mathematics hard at some point were students for whom topics in high school came easily. Of course, we don't mean all topics, as even practicing mathematicians have favorite topics and, well, less favorite topics. One of the editors, David Taylor, still hates factoring polynomials; he was never really good at it, prefers never to do it, and just doesn't like it at all, but working with matrices that have over 1,600 entries and working with their powers to find long-term steady states in Monopoly® comes more easily, and honestly is more fun.

For most, there's a time when the content being studied becomes *hard* and it can be challenging to overcome that. The feeling of understanding everything in class turning into a feeling of not understanding anything can be a shock. What follows are some stories from people who have gone through that exact scenario at some point in their studies; many of these people are currently mathematics professors. If they can struggle with material and make it through to become a professor of the subject themselves, you can too.

1

In the Deep End in Algebra

Deanna Haunsperger

When I was in high school in rural Iowa, I wanted to be a high school teacher of mathematics. My parents had always held education up as the road to success, but since I was a first-generation college student, I had no advice from home regarding college, nor did I know that there were things I didn't know about selecting a college. I chose a small, regional liberal arts college near home in Iowa for my undergraduate degree. Once in college, I knew from the ethos of my environment that I really wanted to be a professor at a liberal arts college.

I naively believed that every college and university had the same curriculum, and that just their location, the enrollments, and class sizes differed. Even though I had taken my high school's first-ever offering of calculus, which included delta-epsilon proofs, my college had me start again in first-semester calculus because they didn't have an appropriate way to place someone higher. I didn't know there were colleges that offered more mathematics classes and taught at a higher level than others. I thought all colleges would have had the same limited offerings mine did. I took all that

was offered, which, at the high end, included one semester of abstract algebra taught at a low level by a visiting faculty member who ended up quitting in the middle of the semester. Real analysis wasn't offered, but at least someone was looking out for me and said I should know some real analysis before graduate school, so I did some independent reading. I also didn't know that I should study for the math subject GREs, and no one told me to. I thought they were like SATs or ACTs, which I hadn't studied for either and did well. I did not do well on the math subject GREs.

I was very fortunate to be accepted to Northwestern University—the graduate program did not require submission of subject GREs—and that's where I went. The graduate student chair was surprised to see how weak my transcript was when I arrived (I was still at that time clueless), and he recommended that I take a year of both real analysis and abstract algebra at the undergraduate level before starting graduate courses. I was devastated; I thought I was as prepared as everyone else. I agreed to a year of undergraduate analysis, but I stubbornly started in the graduate-level abstract algebra course. It was horrible. I had never had a proof-writing course as an undergraduate; I had never had serious homework assignments before, and I just didn't have the background necessary for the material that was being taught. I didn't understand how everyone seemed to know the terms that the professor was using. One of my colleagues was even correcting her at the board, while it was all I could do to try to write down what she was saying so I could read and reread it many times, late into the evening, to try to understand. The anxiety, stress, and feeling of being a failure permeated my life, even to the point of me needing to get two crowns after breaking my teeth from grinding them while I slept.

But I'm stubborn. I knew I needed a PhD so I could have the career I wanted, and I wasn't going to give up. My other math classes were going well, yet I still didn't believe that my problem wasn't in my personal being, but rather in my poor preparation. It took me years to get to a place where I could believe that. Meanwhile, I dug in.

I bought an undergraduate textbook in algebra and started teaching myself algebra from page one. At this point I was weeks behind in my graduate algebra course, trying to learn undergraduate and graduate algebra at the same time. Luckily, my good friend Janice was digging in to master algebra as well. The two of us approached our algebra professor, asking how we could get additional help, and her response was, "Well, graduate school is baptism by fire." The professor threw us in the deep end on our own with a real sink-or-swim philosophy.

At the end of the first quarter, I earned a C in graduate algebra, the lowest grade I had ever received by far and which my colleagues explained was really a so-called "gentleman's C." I had failed. But that was just the first term. It was a year-long course over three quarters, and I was in it for the long haul. However, the second quarter, I also earned a C, but by then I had taught myself most of undergraduate algebra and was working on understanding graduate algebra. Finally, by the third quarter, I was getting caught up on all the material I should have learned years earlier and how to be a graduate student, and I earned an A–. I knew I had finally succeeded when, in the fall of my second year, I passed my algebra prelim on my first attempt. It shocked my algebra professor as well, who had assumed I would disappear over the summer.

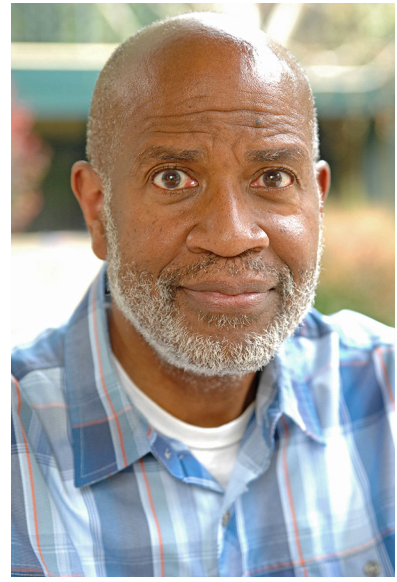
It was my Midwestern roots and my parents' work ethic that enabled me to buckle down and persevere when my path became rocky. I wouldn't wish for anyone the difficulties I had; in fact, the successful Carleton Summer Math Program, an enrichment program for women to give them the information they need to be successful in graduate school, was started by Stephen Kennedy and me to address some of the very issues I faced upon entering graduate school. However, I do think earning my degree and the successes I've had since then in my career are all the sweeter because I know that I had obstacles to overcome to get here. I no longer beat myself up for not having an innate graduate abstract algebra intelligence, but instead I value my grit.

Deanna Haunsperger is a professor of mathematics at Carleton College in Minnesota. Since her own undergraduate days at Simpson College and graduate days at Northwestern University, Deanna has been interested in increasing the number of students who pursue advanced degrees in mathematics. That passion has guided her as a former co-editor for Math Horizons *(the Mathematical Association of America's magazine for undergraduates) and as co-founder and co-director of Carleton's Summer Mathematics Program for Women (a successful, intensive four-week summer program to encourage talented undergraduate women to pursue advanced degrees in the mathematical sciences). She has chaired the MAA's Strategic Planning Committee on Students and the Council on Outreach. Deanna has also served as President of the MAA. Deanna is married to fellow mathematician Steve Kennedy, and together they have two grown children.*

2

The Road Less Traveled?

Lloyd Douglas



Courtesy of Laurie-Jean Sennott

I can't say that I was always good at math. I remember as a child, my father, who had a sixth-grade education, trying to explain decimal-to-fraction conversions to my sister and me. He may as well have been speaking a foreign language. I couldn't understand what decimals had to do with fractions, much less be able to convert one to another. My father eventually got frustrated and gave up trying to explain it to us as we had no clue. However, afterwards, I didn't stop trying to figure out what he was talking about. Finally, a light went off in my head. I tried one case, and it worked. Then I tried several more and they all seemed to work, too. I learned two things from the experience. First, although I can be inspired by others, I learn better by thinking about things myself. Second, if you work hard enough, you can figure out some things—though perhaps not all.

After clearing that hurdle, I excelled in math, at least for a while. I went to an engineering high school in Brooklyn, NY because I thought I wanted to be an engineer. To be honest, I didn't really know what that meant. My

high school was very competitive. Only the top junior high school students in the city were even allowed to take the entrance exam, and only about the top 10% of those were admitted. It was here that I faced my next big math roadblock when studying plane geometry. There were these "non-math" things called theorems and proofs. "Why did I have to learn how to prove something that has already been proven," I asked myself. I struggled big time. It was a feeling similar to when my father was trying to teach us decimal-fraction conversions and just seemed completely alien to me. I made it through the semester, though, and then things returned to "normal" when the second half of plane geometry was applications of the theorems. I told myself that I was glad that I'd never have to prove a theorem again. That couldn't have been further from the truth!

In college, I wasn't sure what my major would be. I've always had broad interests and was dismayed that I had to choose one thing. I delayed the decision for as long as I could and took courses that majors would take in chemistry, German, mathematics, and physics before somehow deciding to major in math. I struggled through advanced calculus because, as the professor said, this was the course where you had to prove the theorems that you used in calculus. So, my nemesis had reared its ugly head again, but it seemed worse this time and I didn't know if I'd get through it. But something caused me to stick it out. After an exam where I didn't particularly do well, my professor called me aside and encouraged me to pursue the discipline because of something she saw in me. I never figured out what she saw in me. She didn't say, and although I was curious, I wasn't going to ask. That simple phrase of her saying that she saw something in me was enough encouragement to keep me plugging away, even though every math course I took after that was proof based.

Despite my struggles as an undergraduate, I decided to apply to graduate school. People told me that graduate school was easier than being an undergraduate because you only took classes in your major. I did relatively well in my first graduate program and earned a master's degree before entering a PhD program elsewhere. I soon decided that I didn't want to be there, so I sought employment. But jobs were hard to come by, so I stayed in graduate school until I finally got a job working for the U.S. Navy as a mathematician. In the meantime, I had been in graduate school long enough to finish my coursework and pass one qualifying exam, but after I left, I never looked back. That job with the Navy was the beginning of a federal career which spanned over 30 years. I spent 23 years at the National Science Foundation, during the last 14 of which I was a program officer in the Division of Math-

ematical Sciences. It was there where I found my niche in the profession: enabling the careers of others. This wasn't exactly the same as the encouragement that I received in college because it wasn't on a one-on-one basis, but what I wanted to do was to be able to provide opportunities for as many people as I could to pursue mathematics and give them the ability to have that one-on-one experience with other mathematicians. My career as a program officer was where I received most of my federal achievement awards. Even now, more than ten years after leaving NSF, people come up to me at conferences and tell me how much I influenced their lives and contributed to their career success. I reflect that if I hadn't been encouraged I wouldn't have been in a position to do so. I am glad to have been able to do so.

Lloyd Douglas is an independent consultant. A Certified Research Administrator, he is the former Associate Director of the Office of Contracts and Grants at the University of North Carolina at Greensboro. Before going to UNCG, he was Assistant to the Vice President for Research at the University of Nevada, Reno. From 1984 to 2008, he worked at the NSF and, while there, he oversaw a large increase in the REU program in the Division of Mathematical Sciences. He also managed the Mathematical Sciences Postdoctoral Fellowships, served on the coordinating committee for NSF's CAREER program, and co-chaired the implementation committee for the NSF's ADVANCE program. He received NSF's Meritorious Service Award in 2007.

3

Help Will Always Be Given at Hogwarts to Those Who Ask for It

Allison Henrich

When I was in third grade, I wanted to be a third-grade teacher; when I was in sixth grade, I wanted to be a sixth-grade teacher; and when I was in high school, I wanted to be a high school teacher. Finally, when I got to college, I settled on the idea of teaching some subject at the college level.

Throughout my first four years of college, I was enamored with philosophy and decided I'd pursue an advanced degree in philosophy to become a professor. I took Epistemology, Philosophy of Science, and Ancient Philosophy, but I got most excited about taking every kind of logic course I could get my hands on. When I was at the University of Washington, the philosophy department offered Advanced Logic, Axiomatic Set Theory, and Computability Theory. These were all gorgeous, fascinating courses! I learned about different sizes of infinity, the axiomatic foundations of the natural numbers, fascinating ideas like Russell's paradox, and I even built a universal Turing machine! Enjoying these courses as much as I did made me think that I might actually like to take more math. I had already

satisfied my math requirement for graduation by taking Calculus 1, but then I decided to take Calculus 2, Calculus 3, Multivariable Calculus, Linear Algebra, and Differential Equations—just for fun. After my Differential Equations final (which was, theoretically, going to be my last math class), my professor followed me into the hallway and asked if I was a math major. When I told him I was just taking the course for fun, he looked shocked and told me I should consider adding a second major in math. This moment of encouragement very likely changed my life.

I declared a second degree in math, and I devised a plan to finish my math degree in two years while absorbing all the philosophy, French, and German that I possibly could. I also quickly realized that I did not actually want to teach philosophy at the college level, but I thought I'd really enjoy teaching math.

The only problem was that math was getting incredibly hard as I got into real analysis, abstract algebra, topology, and geometry. What gives? Math used to be easy! Now, I'd spend hours every day attempting—not terribly successfully—to do my homework. I slept with piles of math books and scribbles of bad proof ideas in bed with me. I dreamt about finding solutions to vexing problems. "Of course! It's just $2+2=4$!" I was disappointed to learn upon waking that the solution was still completely elusive. I could ponder a single problem for hours, getting nowhere, getting discouraged. I'd eventually give up and go to my professor's office hours to beg for help. I distinctly remember spending four hours one evening trying to figure out how to show that one particular sequence converged. When I saw how to do the problem the next day in Professor Mitchell's office hours, I felt like a total idiot. The solution had been right in front of my face! Throughout the five or six courses I took from him, one professor in particular, Steve Mitchell, helped me through all of my mathematical woes. He helped me find solutions to my problems, but he also gave me pep talks and never let me forget how beautiful math is. Even though I no longer felt like I was a natural at math, seeing its beauty and being encouraged by Steve helped me stick with my plan of teaching math to undergraduates.

"What's the least amount of schooling you need to teach college math? A master's degree? Sure! I could probably handle that." By the time I was nearing graduation, I thought that there's no way in hell that I could prove something in math that has never been proven before—math is just getting too hard and I'm not good at it anymore. So I all but abandoned my idea of getting a PhD. Of course, it's much easier to get funding if you say you want to pursue a PhD than if you say you want to pursue a terminal master's degree. So what if I told a little fib and applied to PhD programs with the secret plan to stop at a master's? How much could it hurt? It's the only way I

could afford to pay for graduate school, so I decided I'd give it a try. Luckily, I pulled the wool over Dartmouth's eyes, and they accepted me into their PhD program.

When I got to grad school at Dartmouth, my courses were incredibly hard. But I had already experienced how hard math could be as an undergraduate, so I knew how to work my way through it. All I needed to do was take great notes in class, rewrite them afterwards, spend hours each day on my homework, visit my professors' office hours a few times a week, and study with my friends. Many of my peers had coasted all the way through their undergraduate math degrees, so they were struggling for the first time in these graduate-level courses. It took them a lot longer to figure out how to handle it. But I knew how to struggle. I knew I had good study skills, and I knew how to get help from my professors and friends when I needed it (which was pretty much all the time).

To pass my qualifying exams and earn my master's degree, I just had to apply these same study techniques to qualifier preparation. And it paid off! I passed all four of my oral exams on the first try. Looking back on what I had achieved, I saw how I could take a seemingly impossible task and break it down to tackle it piece by piece. So why couldn't I earn a PhD?

In the end, I kept going. I learned that it doesn't matter whether an understanding of difficult material comes naturally to me or not. What matters is that I know how to respond when things get hard. This frame of mind allowed me to recover when the proof of the main theorem in my thesis fell apart and I had to totally abandon the whole project. After some tears and beers, I found an entirely different topic to learn about and start working in. And although I had to shift the focus of my dissertation in my fourth year of graduate school, I completed an entirely new dissertation and graduated on time in my fifth year. In the end, I got a great job teaching college students—just as I had always wanted.

Allison Henrich is an associate professor of mathematics at Seattle University, where she has been a member of the faculty since 2009. She earned her PhD from Dartmouth College. Allison is passionate about teaching, and she is active in research in knot theory and recreational math as well as the scholarship of teaching and learning. One of the most rewarding activities she engages in as a professor is working with undergraduate researchers. Through knot theory research, Allison mentors students—many of whom are unsure about their career goals—to help them learn what may and may not excite them about a career involving mathematical research. In general, she gets the most enjoyment out of supporting students to do their best work as they learn about the beauty of mathematics. Allison recognizes that she would not be where she is today without the inspiration and encouragement of several of her own undergraduate professors.

4

I Don't Know What I'm Saying
Using Language as a Model for Embracing Mathematical Struggle

Steven Klee

In the spring of 2004, I participated in the Budapest Semesters in Mathematics (BSM) program, which was simultaneously one of the most challenging and most rewarding experiences of my life. My first few weeks in Budapest were an immersion course in confusion. It was a two-part course, consisting of Confusion 101: "Math is harder than you ever thought it was," which was taken concurrently with Confusion 201: "Don't you wish you understood what all these Hungarians are saying?"

The first time I went to the grocery store, I had to point at a pile of sliced deli meat and hold up ten fingers to ask for ten of whatever they would give me. Gracefully I got ten decagrams of slices and not ten kilograms. I was scolded for not weighing my bananas before proceeding to the checkout, and the bottle of water I bought exploded all over my shirt when I opened it because I didn't realize it was carbonated. I struggled to pay for my pass for the metro, to order food in a restaurant, or to understand why (or indeed whether) I was being yelled at in any number of day-to-day interactions. One day, my friend Geoff and I upset a vendor for (apparently) asking for 3.25 kilos of chicken when we wanted three quarters of a kilo.

Communication was challenging and frustrating, but I had gone in expecting it to be difficult. This was my first visit to a non-English-speaking country, so I went with the flow as best I could and very quickly learned to say *Bócsánat, nem beszélek magyarul* (I'm sorry, I don't speak Hungarian). The math, on the other hand, was a different story.

I had struggled with math at various points in my life, but for the most part it had come easily to me. For the better part of the previous six years I had always had one of the top grades in my class. I was used to being "good" at math, so the first week at BSM was a shock. I couldn't do most of the problems on any of my first homework sets. I hadn't taken an introduction to proofs course, so my analysis and topology courses were brutally difficult. I was taking the conjecture and proof class with people who had scored in the top 50 on the Putnam Exam, an exam taken by about 3500 undergraduates across the US and Canada each year on which the average score is around 4 and the modal score 0 out of 120 possible points. I was taking number theory with people who had already taken abstract algebra and Galois theory, and advanced combinatorics from a professor who had written several papers with the great Pál Erdős, a mathematician whose prolific legacy inspired much of modern combinatorics in Hungary and around the world. Everyone seemed to be getting math when I wasn't. I had no intuitive understanding of the axioms of a topological space. I couldn't figure out why everyone kept going on and on about compactness. I couldn't comprehend that the definition of a prime number wasn't just "its only factors are one and itself." It was a tough time for my mathematical self-esteem.

Eventually there were a few small victories. I remember the first time I understood each individual word in the jingle *Tessék vigyázzni, az ajtók záródnak*, on the subway.[1] A few weeks later, I understood when the vendor told me how much my apples cost at the market without having to be shown the price on a calculator. I dropped the topology course so I could focus on my other classes.

After a while, I could tell that I was learning more deeply. I finally understood epsilon-delta proofs after seeing them for the fifth time in my life. I presented a solution to a problem in my conjecture and proofs class and people clapped. I solved an analysis problem about Fourier series that no one else had solved. I adapted to certain norms of Hungarian culture and became more comfortable with norms of mathematical culture, from writ-

[1] *Tessék* is a multipurpose word in Hungarian. It can be translated as "here you go," "beg your pardon?," "what?," "yeah," "ok," or "watch out." In this case, *Tessék vigyázzni* = please watch out; *az* = the, *ajtók* = doors, and *záródnak* = are closing.

ing clearer proofs to understanding fundamental axioms and definitions. We would visit neighboring countries and, after spending days being surrounded by a cacophony of mysterious sounds in a new language, we found comfort in the familiar *Tessék vigyázzni, az ajtók záródnak* when we finally returned to Budapest. Geoff and I were able to compliment a vendor at the market for slicing exactly 150dg of bacon by eyesight, and we all laughed together when we told him he was the "king of the market." One night I had a dream in Hungarian.

I had gone to Budapest knowing that Hungarian would be a difficult language to learn, but I found that I could eventually navigate basic social interactions because of the immersive experience. I came to see that learning Hungarian and learning mathematics aren't so different. In fact, I had been successful in learning the language because I hadn't started with a presupposition that I should be "good" at it. I adapted the same philosophy about math.

I became more comfortable with being confused and more patient with myself. I accepted that math is hard and that it would take time to understand new concepts. The fact that I hadn't been exposed to certain cultural norms did not mean that I was not capable of adapting to them—I just needed time to practice. As a result, I became more confident and more willing to struggle.

My last exam at BSM was my Analysis final. Afterwards, my professor came to me to go over the exam and talk about my grade in the course. After showing me the few mistakes I made on the test he said "Steve, you didn't do very well at the beginning of this class, but now you got an A on the final. I think you learned a lot, so we're going to forget about your performance at the beginning and I'm going to give you an A in the class." My hard work had paid off.

Steven Klee received his BS in mathematics from Valparaiso University in 2005 and his PhD in mathematics from the University of Washington in 2010. After a two-year postdoc at UC Davis, he started at Seattle University in 2012, where he is currently an associate professor of mathematics. In his free time, he enjoys cooking and spending time with his family.

5

A Complex Conundrum

Matthew Pons

Courtesy of North Central College

For most of my life, my success came in the classroom. I enjoyed sports as a kid, but I'm not particularly competitive, so that wasn't my place to shine. I enjoyed the arts, but, for whatever reason, I never invested my time there. I liked learning from books. I liked seeing facts and understanding why things happen. And I was naturally good at remembering these facts. This made me an ideal student throughout most of my educational experience.

But failure eventually came for me. It was the beginning of my second year of graduate school, and before the semester began, I had to take my general exams. These are standard in graduate programs and cover material from undergraduate courses and first-year graduate courses. To say they are intense is an understatement. I took two that fall, one in algebra and one in analysis with two parts, real and complex analysis. I passed the algebra exam and the real analysis portion of the analysis exam but failed the complex analysis exam. Unfortunately, you had to pass both the real and complex parts to pass the analysis exam. And I didn't just fail … I failed

in stupendous fashion (at least, that's how it seemed to me at the time). I had not answered a single question in its entirety! I made progress on each question, but I had no complete solutions. And of course, the fact that you can't continue in a graduate program until you pass these exams added extra heft. This experience had a tremendously negative impact on my confidence and made me question whether being a mathematician was the path for me. As an undergraduate, I had taken complex variables in the fall of my junior year. A few weeks into the semester, I was having difficulty keeping up with the material, but this was mostly due to personal stress. I ended up withdrawing from the course and never retook it. The W it left on my transcript haunted me and made me feel like I had been defeated. And now I had been defeated again.

With a new semester beginning, I couldn't figure out how to process these compounded feelings and the thought of having to retake the exam nauseated me. This went on for about two weeks, but I finally decided that all I could do was try again. I had no interest in admitting defeat. I had no backup plan! That fall, I sat in on the course again and tried to understand my deficiencies, but it was difficult. Something just didn't seem to click, no matter how much time I invested. Some problems seemed like they should be easy, but I could not figure out what was being said when I was shown a solution. It was like my brain had decided not to understand. When the semester was over, I studied on my own, and it did seem to be getting better. Seeing the material for a second time not only helped me to get a better grasp on the theory, but it also allowed me to learn from my previous mistakes. Then the holidays came and ate up a lot of study time. On top of that, I came down with a nasty case of impetigo (a bacterial infection). And the furnace in my apartment blew out. After being away for the holidays, I returned to an apartment that felt like a sauna. My plants were withered and dead. Candles had even melted! But still I studied. I was feeling confident about the material. Then, two days before the exam, I realized I had not reviewed the content for the other half of the exam, the real analysis portion. As I scrambled to refresh my memory, all I could do was hope things would go well.

The complex exam was first. We had three hours to complete six problems. I started the period very confidently, but each time I thought I was making progress on a problem, I hit a roadblock. After an hour and a half, I had not completed a single problem and was in a state of panic. I went to the restroom to splash some water on my face. I remember looking into the mirror quite vividly and I said, "Go back in there and answer those questions!

You can do this." It sounds corny, but it worked. With a little self-confidence, I steadily chipped away at the problems and found my way. I cannot say I figured out all six, but I felt good about five of them, and I had significant progress on the sixth.

The next day, I took the real analysis portion of the exam and felt confident with my performance there as well. Several days later, we got our results, and I had passed. Four months of work had paid off and the graduate advisor told me that I had performed very well, particularly on the complex portion of the exam. She later became my thesis advisor, and under her direction, I grew to love complex analysis. Today, it is one of my primary areas of study.

I know that things might not always end this perfectly. Or it may take much longer than expected to conquer an obstacle. I failed on what felt like a large scale at the time. In my experience, small-scale failures like failing a quiz or exam are equally disappointing because they chip away at our self-confidence. I've learned to keep in mind that it isn't about the scale of the failure or the length of the struggle. It is about the lessons we learn along the way and the resilience we build up in ourselves. In the end, the struggle is most definitely real, but without struggle, there is no reward.

Matthew Pons is a professor of mathematics and chair of the Department of Mathematics and Actuarial Science at North Central College in Naperville, IL, where he has been a faculty member since 2007. He earned his PhD from the University of Virginia in 2007 and his undergraduate degree in 2002 from the University of North Carolina at Asheville. Matthew's favorite professional activity is teaching and his approach to classroom learning has been heavily influenced by the faculty mentors he had as an undergraduate and graduate student. As an instructor, it is not always easy to watch students struggle with the challenges inherent in the study of mathematics. Seeing students overcome these challenges is a constant source of inspiration for Matthew, not only in his teaching but also in his own research. In addition to mathematics, Matthew enjoys cooking, reading (particularly classics and manga), video games, and telling stories.

6

An Accidental Mathematician

Jennifer Quinn

Courtesy of University of Washington Tacoma

It is hard for me to believe that other people find my ideas interesting. To this day, I dislike speaking out in class or faculty meetings. Maybe I worry too much what others will think. I jokingly categorize myself as "the dumbest of the smart kids" and was shocked to find how many of my colleagues felt the same way. They say "fake it until you make it." That applies to almost all of us.

I may be more fortunate than most. I have been encouraged and supported by family, teachers, college professors, my graduate advisor, and faculty colleagues throughout my academic journey. There was never a moment when I felt that I couldn't be a mathematician; it wasn't even on my radar until my early twenties. But there were many days of frustration from the very beginning.

Sixth grade. Mental math quizzes. The teacher would read a question and we were to write down the answer (and only the answer). It was disastrous. I couldn't "see" the problem in my mind's eye and consequently couldn't answer the questions. I felt physically ill on quiz days and wanted nothing

to do with that class. Thankfully this assessment practice did not continue into higher grades.

First semester sophomore year of college. Linear Algebra. I understood everything that was presented in class but had no clue how to do any of it on my own. Yes, it was my first introduction to proof. And there was an annoying freshman in the class who also answered every question with perfection making me doubt myself at every turn. The good news was that I was not alone and the bonds of friendship created through that struggle have stood the test of time.

Junior year of college. Analysis. I had absolutely no intuition for anything having to do with analysis—real or complex. My professors were encouraging and I worked hard to understand. It required creating stacks of index cards with every definition, lemma, theorem, and corollary known to date. Solving problems consisted of reordering cards until I created a path from the given hypotheses to the desired conclusion. Take-home exams were 24-sleepless-hours filled with doubts and self-recriminations. More than two decades later I learned that some of those test problems had previously occurred on Putnam exams and they were hard for everyone, not just me. But that didn't help my confidence at the time.

Graduate school. Qualifying exams. I did not pass the algebra qualifying exam on my first attempt even though my specialization of combinatorics is a subdiscipline of that field. Was I embarrassed? Yes. Did it stop me from pursuing my degree? No. I had worked in industry between college and graduate school and was motivated to avoid another 9-to-5 job. In fact, qualifying exams were not an unusual failure at the time and I took comfort from others in the same situation; we studied together to succeed in the next testing cycle.

My struggles were not deterrents to pursuing mathematics because they mostly occurred prior to the realization that I might want to be a mathematician. I took combinatorics as an elective in the second semester of my senior year to complete a last-minute double major. I loved the problems and had a real intuition for their solutions. If I went to graduate school, (and at that point it was a big if), it would be to study combinatorics. Once I chose to follow that path, there was no turning back even as more struggles followed.

Mathematics chose me. I did not choose mathematics—at least not initially.

What have I learned through my struggles?

First, never compare yourself to others. There will always be someone smarter or more prepared or just better all around. Set a personal goal and

surround yourself with allies who will help you achieve it and promote it. Most importantly, enjoy the journey along the way because the emotional high from achievement lasts only until the next goal is set.

Second, mathematics is not a solitary pursuit. I was never alone and people (whether combating the same issues, local experts, family, or friends) were willing to lend a helping hand. Crazy ideas uttered out loud have more power than realistic ideas kept within the confines of your own thoughts, they spark innovation and real solutions. So build trust within mathematical partnerships and collaborate, collaborate, collaborate.

Finally, dream big and work hard. I earned my PhD and have accomplished more than I ever imagined possible. I'm a full professor, award-winning teacher and author, and have held leadership positions in academic administration and professional associations. I still enjoy dreaming about what comes next for me and my mathematics.

Jennifer Quinn is a professor of mathematics at the University of Washington Tacoma. She earned her BA, MS, and PhD from Williams College, the University of Illinois at Chicago, and the University of Wisconsin, respectively. She has held many positions of national leadership in mathematics including Executive Director of the Association for Women in Mathematics, co-editor of Math Horizons*, and Second Vice President of the MAA, and Officer-at-Large on the MAA Board of Directors. She received a Haimo Award for Distinguished College or University Teaching and a Beckenbach Book award for* Proofs That Really Count: The Art of Combinatorial Proof*, co-authored with Arthur Benjamin. She is writing and illustrating a young adult novel described as "Griffin and Sabine meet Hugo Cabret and do some math." She lives in Tacoma, Washington with her husband Mark Martin, a microbial geneticist (not a racecar driver), and two teenage sons Anson and Zachary.*

7

Nowhere to Go But Up

Lola Thompson

One month after turning 21, I left behind a highly anticipated summer of living with my best friend in a swanky condo in Chicago and moved into a spartan dormitory in Columbus, Ohio, one that I would share with a bunch of 15-year-olds under a strict "no alcohol" policy. After nearly failing abstract algebra (a last-minute "withdraw" from a sympathetic postdoc was my saving grace), I was hoping to find redemption in a math camp aimed at high school students.

It took me six hours to drive from Chicago to Columbus, my first solo road trip, and the endless cornfields only seemed to exacerbate my feelings of isolation. To pass the time, I found myself rehearsing how I would explain my situation to the high school kids who also would now be my peers. My story converged on something like this:

> My high school experience was not like yours. I went to a performing arts high school and spent my days taking courses on Opera Performance and Historical Social Dance instead of trigonometry and pre-calculus. My math classes were relegated to the "math trailer" because the building was short on space. This didn't bother me when I dreamed of becoming the next Maria Callas, but it hurt when

I decided that I wanted to attend a "normal" college instead of a music conservatory.

I spent my first three years at the University of Chicago as an economics major. I worked hard to convince others, and myself, that I was studying economics because I loved the subject, and not because I was afraid that I wasn't smart enough to major in math. I found myself inventing all sorts of reasons to justify why I really needed to take courses such as real analysis, discrete math, and abstract algebra as a future economist. I started skipping my economics classes so that I could attend special lectures in the math department. I played so many games of *Set* with the Math Club that I nearly got elected Math Club President.

Towards the end of my third year of college, my undergraduate adviser informed me that I had accumulated enough credits to graduate a full year early. Rather than feeling happy, I was devastated. I had this sinking feeling that I was graduating with the wrong degree. I finally admitted to myself that math was the subject that I had loved all along. I marched into the mathematics department and proudly informed the Director of Undergraduate Studies that (1) I was declaring a math major, (2) I planned to go to graduate school in math, and (3) I would need to withdraw from abstract algebra because I was failing. She did not seem impressed, but she also didn't want to discourage me outright. She suggested that I spend the summer trying to "catch up," whatever that meant.

I realized that my poor performance in abstract algebra largely stemmed from my weak proof-writing skills. I didn't understand when my proofs were "rigorous" and when they weren't. I didn't understand whether certain details were "necessary" or "extraneous." When comparing my proofs with the professors' homework solutions, I couldn't distinguish whether my arguments were flawed or whether they merely presented a different approach.

I sought out the strongest math majors in my classes and asked them how they learned to write proofs. Several of them swore by these summer math camps for high school students. I wrote a desperate last-minute e-mail to the director of the Ross Mathematics Program and he took mercy on me. He agreed to let me attend the program, but only under the condition that I would follow the same rules as the high school students.

So, here I was, arriving in Columbus, embarking on a summer of early curfews and mandatory lecture attendance. I told my story to incredulous

high school students who were already dancing circles around me mathematically, teenagers who would grow up to win prestigious prizes in mathematics and who would certainly never know what it feels like to fail a college math course.

I gave up an exciting summer on the blind faith that a high school math camp would somehow change my life. And it did.

I returned to college at the end of the summer and told the Director of Undergraduate Studies that I wanted to sign up for the honors-level abstract algebra sequence. She took one look at the "W" on my transcript from the non-honors-level version that I took during the previous term and asked "are you sure?"

I earned a B– in that first honors-level math course, which may not sound like much, but it was a huge victory for me. My grades increased in every subsequent term, as did my confidence level. When it came time to apply to graduate programs, to my surprise, several took a chance on admitting me, in spite of my less-than-stellar transcript. Graduate school was challenging, and there were times when I had reason to worry about failing out. Each time, I would think back to that lonely drive across the Midwest and remember how far I had come.

It can be useful knowing that the lowest point of your mathematical career has already happened. From that point onwards, there's nowhere to go but up.

Lola Thompson is an assistant professor of mathematics at Oberlin College. She earned a BS in mathematics and a BA in economics from the University of Chicago. She went on to receive a PhD in mathematics from Dartmouth College under the direction of Carl Pomerance. She spent a year as a postdoctoral researcher at the University of Georgia, and has spent recent semesters as a visiting scholar at the Max Planck Institute for Mathematics and at the Mathematical Sciences Research Institute. Her research is in analytic number theory. In her spare time, she trains and performs as an aerial acrobat. She is also an avid traveler.

8

Hitting the Wall

Laura Taalman

Math was easy for me, until suddenly it wasn't. I suspect this is a transition that many people go through, some having a difficult time in high school and others hitting the wall in graduate school when they have to start doing serious work to succeed. I think sometimes people who hit that wall too late in their journey have trouble dealing with it because they never had to develop the necessary study habits and capacity for successfully handling "being stuck" without giving up. I was fortunate in that I hit this wall the minute I went to college and was somehow able to handle transitioning from the high school kid who was best at math to the dumbest, least-prepared kid in her calculus class.

I grew up in a small town of fewer than a thousand people, and at my high school, there were no calculus courses. I knew that I wanted to do math in college and that I wanted to test into the honors calculus sequence at the University of Chicago, where I was to be an undergraduate. So, I took a quick-and-dirty computational calculus course over the summer at a local college. Somehow, this was enough to allow me to pass the placement exam that put me in the honors sequence, so that's where I started in college. Of course, many of the other math majors passed out of calculus entirely and were starting out with honors real analysis, so for the major at Chicago, I was actually starting out on one of the bottom rungs.

In this honors calculus sequence, we used the book by Spivak—an insanely theoretical and difficult treatment of the subject. At the time it was a real shock to me. It was my first introduction to real mathematical thinking and proof. I had to work easily ten times as hard as everyone else in the class, literally. I remember spending every day working for hours to try to figure out the material and setting aside my entire Sunday—from when I got up to when I went to bed—every week to work just on this one calculus class. And even so, I was always the student with the dumbest questions and the most confused-looking face. I was barely hanging on for each homework assignment.

Having to work so hard that semester and develop a thick skin for feeling stupid all the time in a math course really helped me in my career. I don't think mathematics was ever "easy" for me again after I got to college; it was always a struggle. College math courses were all hard, graduate school was hard, and researching and writing my thesis was hard, too. But at each step, it was a struggle that I loved working through. I didn't mind being stuck and feeling dumb. I knew I could get through it if I kept plugging away. In today's language, I would say that I was lucky to have formed a "growth mindset" about learning math—I was willing to work on hard problems to find success—rather than a "fixed mindset," where I judged myself harshly when I didn't know something. To this day, I still benefit from this mindset, and I've basically made a career out of trying new things that I don't know anything about. It's in the process of making mistakes and figuring out how to make progress where the real fun begins.

Laura Taalman is a professor of mathematics at James Madison University whose research has included algebraic geometry, knot theory, and games. Laura also publishes calculus textbooks and Sudoku puzzle books, blogs at Hacktastic and Shapeways, and designs and shares hundreds of models with the 3D printing community, where she is known as "mathgrrl." She consults for 3D printing companies as an "expert amateur," a completely real thing which here means "a person who is good at doing things that they aren't particularly good at." Laura is a Project NExT Fellow, a recipient of the Alder Award, Trevor Evans Award, and SCHEV Outstanding Faculty Award, and has been featured on Thingiverse, Adafruit, and Science Friday.

9

To Algebra or Not to Algebra

Jacqueline Jensen-Vallin

Courtesy of Robert W. Vallin

As an undergraduate, I was a psychology major until what should have been my last semester. I had extra space in my schedule that semester, and I decided to take Calculus III, having not taken any mathematics courses for about two years. I loved it! It was like coming home! So, I took another year and completed a math major, graduating with dual degrees in mathematics and psychology.

During that crazy undergraduate year, I took courses in the introduction to proofs, complex analysis, linear algebra, differential equations, probability, and, of course, both analysis and algebra. The analysis course was out of Ross's undergraduate text and I loved it—the whole thing felt like a puzzle. Modern Algebra was the same—it was fun to play with abstract objects that had familiar properties, and I felt like my proof skills grew in this course. So, I did what many others have done when they were graduating and didn't know what else to do—I applied to graduate school.

When I started graduate school at the University of Oregon, I was equally interested in algebra and analysis (Ross taught there, but he was retiring when I arrived), and I was excited to pursue my PhD in either discipline. I was really saddened when I struggled with graduate algebra, even though I

started in the cross-listed undergraduate/first-year graduate sequence and then took the qualifying exam-level sequence my second year. Even in my second year, I was still struggling to understand and prove algebraic theorems. Fortunately, I was doing okay in analysis, and I loved topology (which would later become my dissertation area).

During that second year, I decided that I would study for course exams by doing old qualifying exam problems. So, I did. I worked old qualifying exam problems in algebra for two weeks before winter quarter finals. When I got to that final exam, though, I couldn't even state the definitions and theorems, I had no entry point for the true/false problems, and I couldn't begin any of the proofs. I flipped all the way through the exam looking for a place to start. I couldn't find one, so I started laughing. (At the time, it seemed better than crying.) I sat through the exam and vaguely remember working on some pieces.

After the exam was graded and scores had been posted, I went by the professor's office to talk to him about my chances of success and whether I should try to take the qualifying exam in algebra or in analysis. Analysis was rumored to be the most difficult. He said, "Ah, yes. I did notice that you got the lowest score [in the class] on the final last quarter. When considering which qualifying exams to take, you should maximize your chance of success."

Required to only take two qualifying exams, I took analysis and topology. I thought I might be done with algebra permanently at that point, but my dissertation proved me wrong. It was about a topic in algebraic topology, using algebra as a tool for understanding topological concepts. I loved algebra as a tool, but I didn't understand it well enough at the time to appreciate it for its own sake.

I don't think I understood what changed in my perception of algebra—from when I enjoyed it as an undergraduate to my struggles with the subject as a graduate student—until I started teaching algebra as a faculty member. With the benefit of hindsight, I understand that I wasn't able to see the connections between the ideas clearly enough to understand the topic. It required taking several steps away from the subject and getting comfortable with algebra as a tool before I could engage with it in its own universe. Seeing all that algebra could offer to many areas (topology and geometry, in particular), I became more interested in it as its own field.

I now teach undergraduate modern algebra fairly regularly. As a matter of fact, a well-known algebraist got quite a chuckle out of the fact that I was teaching the subject in a bridge-program for students entering graduate

school, given my previous experiences. The first time or two that I taught the class, it was a struggle to be able to see the overarching story of the material well enough to get students to see the big picture. Eventually, though, the story in my head (and, hopefully, my students' heads!) evolved. All I needed to do was to think of the pieces as part of a large puzzle and to emphasize how they fit together. Once I was able to do this, my struggles with algebra faded away.

Jacqueline Jensen-Vallin earned her PhD in low-dimensional topology from the University of Oregon in 2002. She is now a tenured associate professor at Lamar University in Beaumont, TX. In 2004, she founded the Texas Undergraduate Mathematics Conference (which has now become a regional meeting at a rotating venue, and is about to have its 14th meeting). She has successfully engaged undergraduate students in research, and has made it a priority to involve students in the larger mathematical community by taking them to local, regional, and national conferences. She is currently the Director of the First Year Mathematics Experience at Lamar University and editor of MAA FOCUS*, the newsmagazine of the Mathematical Association of America. In addition to her mathematical career, she is a knitter and a dance mom to her twins.*

10

The Unnecessary Struggle of Self-Mandated Isolation

Alicia Prieto-Langarica

Graduate school is hard. Most people will agree with that statement. However, what makes graduate school hard can vary from student to student. I was very fortunate to be immersed in mathematics from a very young age. I was a participant in the Mathematical Olympiad since the age of thirteen. Throughout my time at the Olympiads, I was exposed to formal mathematics, the kind that one learns in most undergraduate programs. The Math Olympiad training sessions were demanding and many times very hard. After high school, I was admitted to one of the most intensive programs in mathematics in my home country of Mexico. The kinds of classes I was taking there look a lot more like graduate-level classes in many other institutions. So when making the transition to a mid-level PhD program, everything seemed to indicate that the struggle had passed.

I was lucky to be able to pass the first two preliminary exams before starting my PhD program, which allowed me to start doing research during my first semester for the first time in my career. Research is so different from classes. During my first research meeting, my advisor and his collaborator

started describing a program that would act as a simulation for a biological system. They talked among themselves and discussed what they expected the program to do and how the simulations should look. Then my advisor turned to me and asked: Do you have any questions? I realized then that *I was supposed to come up with this program!*

Since I had never done research, I had a million questions, but I asked nothing. I nodded and went to my office. It was a Wednesday. In my mind, I convinced myself that research was like class, so when given an assignment, one has a week to complete it. That's how it works, right? The only problem was that I had never programmed. I wasn't even sure what 'Matlab' looked like or where to find it. So, as a lot of people my age were starting to do at that time, I asked Google. I did not sleep too much that week. I spent 12–14 hours every day in front of the computer trying to teach myself how to program so I could turn in my 'assignment' on time. I questioned all my life choices and wondered whether I was cut out for this.

I tried and tried and by the end of the weekend, I had nothing good. Since I had taken this as an assignment, I assumed I could not ask anyone for help and that my advisor had assigned this work because I was expected to be able to do it. You would think that since I was doing well in my classes I would not question my capacity, but you would be wrong. Impostor syndrome has wonderful ways of sneaking up on you when you least need it. It was a very long and short week. I cried a lot and worked a lot. I made a lot of mistakes that week, but none greater than isolating myself. Neither my friends, professor, family, nor my then-boyfriend knew I was going through this. I was too ashamed to admit that this was hard, and I was struggling.

By the next Wednesday meeting, I had a very terrible version of what my advisor and collaborator had been describing—pretty much a cartoon of what they wanted. I was so ashamed and nervous to show the program to my advisor. I was sure he was going to drop me as a student. Much to my surprise, when I presented my progress, they were both impressed and told me what I could change to make it better.

I have learned since then that research is not like a class and that research progress is just that: progress. It's not an assignment that you are supposed to finish from week to week. But more importantly than that, I have learned to ask for help, to admit when I am struggling, and to lean on my professors and peers. Because—you want to know a secret? Everybody struggles. Even when you see them doing well in class, or publishing papers, or succeeding in any way, that success was most probably built on tears and sweat.

Currently all my research projects are collaborations. Even most of my undergraduate student research happens in groups of students that I advise.

I work with a wide variety of researchers and students in many different disciplines. We take advantage of each others' strengths to make up for each others' weaknesses. Many of my collaborators have also become my friends and they advise me in other areas of my professional life, such as teaching, mentoring students, and university service. I am (mostly) not ashamed of asking for help now or admitting that I don't understand something. My collaborators make me a better researcher, and I hope that my students learn about the power of collaboration as I have.

Alicia Prieto-Langarica is an associate professor in the Department of Mathematics and Statistics at Youngstown State University. She received her undergraduate degree in applied mathematics from the University of Texas at Dallas in 2008 and her PhD from the University of Texas at Arlington in 2012. Alicia's research is in the intersection of mathematics and biology, specifically problems related to the medical field. Recently she has been interested in learning data science techniques and how they can help understand problems in the social sciences.

11

The Struggle of Qualifying Exams

Alejandra Alvarado

In my undergraduate years, I always did well enough to maintain a 3.5 GPA as long as I did all of the assigned homework problems, studied, and worked with other students. I enjoyed the work and the challenge, and I assumed it would be the same for mathematics graduate school. With this idea in mind, I relocated from the Bay Area to Arizona, where I could afford to attend graduate school while raising my daughter in a home with a backyard and a dog, something I only imagined as a child. Soon after, my mother retired and joined us. I was told that graduate school would take about five years. I imagined that when I finished, my family could afford to return home. What I could not have predicted was just how difficult that would be. Almost as soon as I left our home in California, the cost of living skyrocketed there. I also quickly realized that going to graduate school was very different than pursuing an undergraduate degree.

When graduate school began, all of the incoming students were placed in the same three core graduate courses. These courses were the basis for

our qualifying exams. The problem was that I did not have the study skills to succeed, and I lacked key prerequisites. Feeling demoralized, I eventually dropped one of the three courses and switched to a prerequisite course for a second course the following semester. By the end of the first year, I was already behind most of my peers.

During this time, graduate students at my institution had two attempts to pass each of the three exams in order to become a PhD candidate. By the end of the second year, I passed one, semi-passed the second one, and failed the third. This meant that I would be leaving the program with a master's degree. I would have been ecstatic to earn an MS degree in mathematics if that had been my original intention, but I had moved my daughter away from family so that I could get my PhD! It was very tempting to just leave without my doctorate, especially since I already felt very guilty for spending so much time away from my daughter while at school. What kind of single parent would be selfish enough to continue working on a doctorate in my situation? But I was devastated. It felt like someone was telling me that I was not smart enough, and I needed to prove them wrong.

Because of my parents' lack of education, a part of me felt like I had to make up for all they had missed. I also felt like I needed to show my daughter that I could do it. Thankfully, I found a support network. The summer after my first year of graduate school, I participated in the EDGE (Enhancing Diversity in Graduate Education) program, which provided me with a mentoring network of women in mathematics (and a few men). Because I did not want to move my daughter again, I stayed in the same town and applied to a graduate program that was two hours away. I was accepted, and the program accepted the qualifying exam that I had passed. By the end of my first semester, I had passed the remaining qualifying exam, and I was finally a PhD candidate. I was lucky in two respects. My new institution was willing to work with me so that the classes I taught would be on the same days as my own classes, making my long commute more bearable. I also found someone who was willing to be my advisor. After eight long years, I was awarded a doctorate degree in mathematics. My parents and daughter were there to witness the event, one of the best days of my life.

The hardest and most demoralizing part for me throughout my academic journey has been taking the PhD qualifying exams. I tried to jump from applied to theoretical mathematics, not realizing that I did not have the necessary background knowledge until it was too late. But what I learned was to not let one graduate program define me. After sharing my story with others, I quickly learned that I was not the only one to have an experience like this.

For various reasons, some graduate programs may not be the right fit for a particular person, and that is perfectly okay. For every type of student, there is a program out there that provides the right kind of support needed to be successful.

Alejandra Alvarado received her PhD in mathematics from Arizona State University. She is currently working at the U.S. Navy in harmonic analysis, but her research interests also lie in number theory. Her most important achievement is obtaining a PhD, while raising a hard-working and conscientious daughter.

II

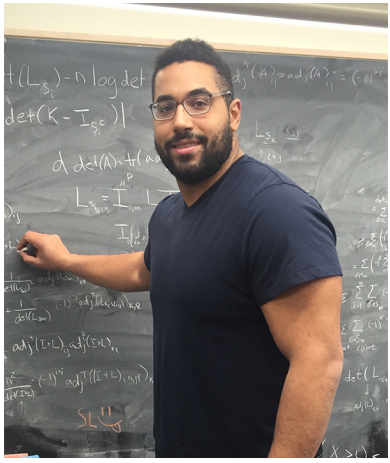
Who Are These People? Do I Even Belong?

If your parents went to college, ask them to close their eyes and describe for you their memories of mathematics in college; chances are, their professors were white and male. Women and minorities have long been underrepresented groups in the discipline of mathematics, but that is *starting* to change. Mathematics is for everyone, and we all want to see more equal representation of race, gender, ethnicity, sexual orientation, and so on, in our colleagues and mathematics friends.

12

I Am a Black Mathematician

John Urschel



Courtesy of John Urschel

I am an outlier. And in my opinion, it has nothing to do with what I have done as an athlete.

I am a black mathematician.

I am an African-American male raised by a single mother. Those facts make me an anomaly, statistically unlikely to love math, let alone to be a doctoral candidate at MIT.

My mother's parents were born and raised in the South, during a time when schools were segregated. My grandmother's classroom was one big room located in the basement of their church, led by a teacher who taught all grades. She only completed the eighth grade, because that was the highest level taught where she lived in South Carolina. My grandfather, as the oldest of his siblings, had to drop out of school to find work to help support his family in Alabama. He eventually got his high school diploma at the age of 25. They both left home and moved to the North, where they became blue-collar workers. Although they possessed only very basic educational skills, they believed that a better education was essential for their children.

My mother attended public schools where the goal was just to get by, not to expand a student's mind. Her high school had high dropout and teen

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pregnancy rates. Fortunately, her math teachers recognized that she had an unusual aptitude and placed her in classes above her grade level. By the time she entered her senior year, she had taken all of the available math classes her school had to offer. Her math teacher, Mr. Stern, took the initiative to enroll her in a college calculus class and arranged for the school to pay her tuition. She was valedictorian of her graduating class. Even so, her guidance counselor encouraged her to become a secretary. Instead of following a path in mathematics, she attended university and became a nurse. Still, her love of math never died.

When I was a young child, walking down the street or riding in the car, she and I would frequently play games. Every week, we had family night, where we would play board games. I thought we were just having fun. I never saw it as learning, but learning I was. My mom would buy me countless educational games and books along with whatever action figure was popular at the time. She gave me a balance of education, fun, and sports.

As early as the first grade, my mom was contacted by my teacher and informed that she believed I had problems "processing," which was a politically correct way of saying that I was unable to grasp the material. The teacher's suggestion was to test me so that they could place me in a class where the material was not so strenuous. As one of my biggest advocates, my mother agreed that I should be tested, because she had no doubt I knew the material. At home, I was an avid reader and devoured algebraic math books faster than my mom could get me new ones. After the teacher saw my result on the test, the teacher suggested that I be moved up a grade level. My mom said no. From her meetings, my mom believed that my teacher saw me as a child from a single-parent household and viewed my shyness as a sign that I was a typical minority child unable to keep up in a classroom setting. My mom withdrew me from the school and looked for educational environments that would not prejudge my abilities based on the color of my skin. She did not want me labeled as a "lesser-than."

In that respect she succeeded—so much so that I can say I have never in my life felt that the color of my skin has ever affected my math, nor how I have viewed myself as being perceived. In many ways, my experience and view of the world as an African-American is disjoint from my mother's. Where she sees racism, I often see fairness. Where I see a struggling student, she sees a minority who has been implicitly told their whole life that they are "lesser-than." Where she strongly supports communities that unite minorities in a given field, I cannot help but wonder if they come at the cost of creating further division.

I sometimes struggle to reconcile my experience with her worldview

—and also with the realities of the field. I know that the color of skin has nothing to do with the ability to do mathematics, and yet when I look at the top mathematics departments in the U.S., I cannot help but notice that MIT, Princeton, Stanford, Harvard, Caltech (and too many others to list) do not have a single African-American professor. I am not sure it is their job to make sure that they have one if there is not sufficient talent, which leads me to believe the talent is not there at that level. Since it is ridiculous to think that all of the most brilliant mathematical minds born in the U.S. are Caucasian, this leaves us with the sad truth that talented African-Americans are being left behind.

The optimist in me says that change has already occurred, and it will be more and more apparent as time goes on. But sometimes, I find too much truth in my mother's sentiments. Sometimes, I find myself meeting with young African-American would-be mathematicians, hearing them ask how I have managed to get to where I am, and watching them hold back tears when talking about being behind or feeling like they cannot succeed because they do not have the background that the "elite" young talent in their classes have had. It is a sobering experience, and I cannot help but feel a sense of privilege for being unable to relate to it personally.

As an outlier, I have a responsibility to set a good example for young people everywhere who have mathematical talent but may feel like they cannot succeed because they do not look like those who have succeeded before. I have a responsibility to succeed, not just for myself, but for my mother, my grandparents, and every minority who feels like the field is closed to them. I am all too aware that as a mathematician, the color of my skin means both nothing and all too much.

John Urschel is a PhD student in mathematics at MIT and a retired NFL offensive lineman. He completed his bachelor's and master's degrees in mathematics at Penn State, while playing football for the Nittany Lions. Upon graduating from Penn State, John was the recipient of both the William V. Campbell Trophy, commonly referred to as the "Academic Heisman," and the Sullivan Award, presented to the most outstanding athlete in the United States. John was selected in the fifth round of the 2014 NFL draft by the Baltimore Ravens. The following year, he was accepted into the PhD program in mathematics at MIT. After his third season in the NFL and second year at MIT, John retired from professional football, and focused on his research in graph theory and machine learning full time. He is currently in the process of finishing his PhD under the supervision of Professor Michel Goemans. In his free time, he likes kickboxing, playing chess, and spending time with his daughter Joanna.

13

Cold, Austere, or Queer

Autumn Kent

There is an idea that our identity has no bearing on our work. That our race, our gender, our disability status, our class, our sexuality, our mental health, has no impact on our professional lives, our productivity, our work. We're just studying math, and all that other stuff is irrelevant.

Well, I'm queer. And it has certainly affected my professional life and my work. I've struggled with depression and alcoholism and that affects my work, too—graduate students suffer disproportionately from depression and we should talk more about mental health in our profession. There is no mathematics 'cold and austere' that keeps me here on earth. I come to mathematics carrying all of my humanity. My relationship with it passes through my experience. It affects my view of my world and I bring to it my own perceptions. I can't relate to Bertrand Russell's notion that mathematics is beyond humanity. Without us it is nothing.

As a trans woman who came out late in her career, I am in a unique position. I have seen firsthand how men behave when women aren't around and how their behavior changes when a woman arrives. I've been told sexist,

homophobic, and transphobic jokes, and I am acutely aware that these jokes are (mostly) hidden from me now that I am out. I have gone from hearing about sexual harassment to being its object.

Throughout my career, it has seemed that the men were having more fun when the women weren't around. You could simply feel the atmosphere change as women came and went. I knew that I would lose something professionally if I were truly myself. There is a certain professional camaraderie available to cisgender heterosexual men that is not available to the rest of us. To my shame I probably contributed to this atmosphere to protect myself. I sacrificed my identity and my happiness to gain professional footholds. Trans women are told that we've enjoyed male privilege by getting ahead being read male. But it doesn't feel like a privilege to trade material gain for one's soul.

And some of the most influential men in these professional circles have in fact been the most supportive of me after coming out. I am deeply appreciative. But I wouldn't have guessed that when I was a student, or a postdoc. I couldn't have been sure. So I kept avoiding certain romantic relationships because they were queer, because I might lose my position in my professional social hierarchy. I kept my masculine disguise. I kept hiding.

And even now I can't do the calculus to determine where I'd be if I'd been myself sooner. I know to some extent what I've lost personally. But I don't know where I'd be professionally if I hadn't taken the path I did. In a queer-friendly culture, I'd be much further along. I lament lost theorems. In the culture I was given? I don't know.

This past year Evelyn Lamb interviewed me for her blog at *Scientific American*. Because of that interview, I get a lot of letters from young queer scientists. They get hope for their professional futures just knowing I exist. Imagine that. Imagine not knowing anyone like you in your field. Imagine not even *hearing* of anyone like you. I have an impact on queer students in my department by just showing up visibly queer. If I'm here they aren't alone. They have an outspoken trans woman professor—someone to come to. That doesn't sound like much but it's big. It's not just that we don't have queer role models. We don't even have queer examples.

When I was a graduate student, I knew one gay graduate student, one gay postdoc at another institution, and no other queer research-active mathematicians. I envied, even resented, the graduate student for being out and prosperous mathematically. He seemed to be doing fine—why couldn't I do that?! The pressure I felt to hide made me resent my only fellow traveller! Eventually I'd hear of one or two, here and there, maybe him, maybe her. I'd meet one, maybe two. Even now it is mostly word of mouth.

We aren't as visible as young people need us to be. They still struggle. They still send me letters saying that they are glad I am here, that they feel less alone. It's still a boys' club and people are still hiding, still guarding their souls.

Autumn Kent grew up in Goldsboro, NC and earned a BA in mathematics and literature from the University of North Carolina at Asheville in 1999. After going to college planning to be a high school English teacher, Autumn decided to focus on mathematics after the second year. After graduating, she taught at a community college before going to the University of Texas at Austin for graduate school and obtained her PhD in 2006 with Cameron Gordon as her advisor. After Texas, Autumn spent four years at Brown as an NSF postdoc and Tamarkin Assistant Professor. In 2010 she went to the University of Wisconsin, where she is now an associate professor. Autumn lives in Madison, WI with her partner and their two daughters. She enjoys cooking, pencil drawing, taking photos, and writing short poems.

14

A View of Mathematics from Behind the Veil

Robin Wilson

Courtesy of Tom Zasadzinski/Cal Poly Pomona

If we think of mathematics as a neutral science that is free of bias, it might be hard to comprehend how there is any other explanation for some groups' lack of success in mathematics other than the fact that they are less capable. But when we investigate how privilege plays out over the course of history, over one's lifetime, and over the course of one's day, we can see how one can be placed in a position to either access or be denied access to an equal chance at participation in society. And societal participation includes participation in the mathematical community. In her 2017 *Journal for Research in Mathematics Education* article "The Culture of Exclusion in Mathematics Education and its Persistence in Equity Oriented Teaching," Nicole Louie describes what she calls the "culture of exclusion" in the mathematics classroom:

> *The restrictive and hierarchical culture that has historically dominated American mathematics education limits all students' access to rich and meaningful mathematics learning experiences and further limits*

many students' opportunities to develop identities as mathematically capable learners and thinkers.

One of my first memorable experiences with the culture of exclusion in the mathematical sciences happened in my transition from middle school to high school. My mother, being the proactive mom that she was, engaged me early on in extracurricular activities around mathematics and science. For instance, she enrolled me in a science-themed summer program in elementary school. Later, when I was in middle school, she got me involved in the Mathematics, Engineering, Science Achievement (MESA) Program. The summer after I took pre-algebra in eighth grade, my mom placed me in a self-paced algebra class that was offered at a local college and taught by a college instructor. I struggled through the course the entire summer and suffered the experience of always being the slowest one, but I survived the long days in class and the long bus rides home. When I went to my high school for the first time to meet my guidance counselor, who happened to be a middle-aged white person, he looked first at me, then at my record, and placed me into pre-algebra again. Despite my summer spent learning algebra, he convinced me that it was in my own best interest to repeat the pre-algebra course. My mother, on the other hand, upon learning about my schedule, marched up to the school the next day to demand that I be placed in the appropriate mathematics class—algebra.

It wasn't until I was an adult that I was able to reflect on the significance of this experience. If I hadn't been given the opportunity to take algebra as a freshman, I wouldn't have been able to take calculus in high school, and I'm not sure where I would be today. I'm sure it's possible that I would still have gone on to become a mathematician, but I would have followed a different path with more obstacles. What's of much bigger concern to me, though, is the realization that there have been many other students, like me, who were placed in mathematics courses below their ability level by this same counselor or by other staff in similar positions of power. These gatekeepers were unintentionally (or perhaps intentionally) underserving the education of so many students. Multiply that number across all the high schools in my home town of Sacramento, the state of California, and the country, and the impact is staggering.

My high school was a public one and was exposed to its share of violence; more than a handful of my peers were victims of gun violence and the prison-industrial complex. For these students, having greater access to mathematics early on could have made a very real difference in their lives. For many students today, the issue of access to a quality mathematics instruction can literally mean life or death.

I have a very vivid memory of another incident that must have happened around my junior year of high school. I was at my school on a weekend, and when I walked past the cafeteria, I was surprised to find it full of people. I stopped to look through the window—I was trying to figure out what was going on inside. I didn't recognize many of the people. I noticed that the students didn't really reflect the entire demographic of our school, which was very diverse. After asking someone what was going on, I found out that it was a math competition! So, there I was, literally on the outside looking in. First, I wondered why I hadn't been invited. Then, I felt glad that I wasn't invited because I didn't think I'd fit in or perform well anyway. It's interesting looking back on that experience from the perspective of a student who also may have been capable but was not invited into the math community at my high school. In addition to being denied access to a mathematical enrichment activity, I was also denied the opportunity to network and build mathematical relationships with other students and teachers from across the metro area. What a perfect example of an experience in which a student may have been perfectly capable of performing well or excelling in a specific math community but was not invited into the room to even find out.

As an undergraduate, I was not invited into the math community by any students or faculty in the math department at my own university, but I was included in the emerging scholars program community which operated in a different space. It was outside of the department, but at the same time, it provided a "safe passage" that we could take to get in and out of the math department space. Almost no one in the math department (except for those in the emerging scholars program, which was outside the math department) ever expressed the slightest interest or confidence in me or my abilities until the last week of my senior year when I handed in my honors thesis project. Then, almost as an afterthought, my advisor suggested I might be capable of succeeding in a graduate program. Looking back, I find it puzzling that I had never even heard of the Putnam exam until my fourth year of graduate school, and I probably learned about the Math Olympiad around the same time. I didn't even know colleges had math clubs for students until I became a faculty member. Instead, my "math club" was the peer group that I formed in the emerging scholars program, the community of role models and mentors that I met through attending the NAM Undergraduate MATHFest conferences, and the mathematicians that I discovered on my own by spending lots of time on the Mathematicians of the African Diaspora website in order to find my own role models to look up to.

Things turned out all right for me in the end, but I wonder how different things might have been if I had been exposed to the same opportunities that

many of my peers and classmates in high school and college had access to. In some ways, it's as if my experience in mathematics has taken place behind a "veil," to borrow a phrase from W. E. B. Du Bois as it was articulated in his 1903 book *Souls of Black Folk*. For Du Bois, the veil is a reference to the experience of blacks in America in which there is a world that they can see and are in many ways a part of but cannot access in the same way as their white American counterparts. The veil takes many forms. It appears as the metaphor of the glass ceiling for women and the bamboo ceiling for Asian Americans. As Howard Winant explains in his 2004 article *The New Politics of Race: Globalism, Difference, Justice*, "For Du Bois the veil not only confined and excluded black people, but also protected them from at least some forms of white violence and domination." He goes on to say that as a consequence of this double-consciousness that is imposed by the presence of the veil, "the veil not only divides the individual self; it also fissures the community, nation, and society as a whole." This metaphor plays out in the mathematical sciences as a fissure in the mathematical community that also passes beyond the community into the discipline itself.

For me, the repercussions of these experiences have been profound. My identity as a mathematician, how I see my role as a faculty member, how I relate to students, and how I relate to and socialize with others in the field are all shaped by these experiences. Now, as a full professor with little left to prove, I still find myself wondering from time to time what it would be like to experience a bit more entitlement around mathematics. I am still unsure of where I belong in relation to many of my peers. The impostor syndrome is still very real. And I often wonder what it would feel like to be able to participate in the discipline without it having to be so much of a struggle.

Robin Wilson is a professor of mathematics at California State Polytechnic University, Pomona. A product of the public school system in Sacramento, CA, he attended UC Berkeley where he developed a passion for teaching mathematics as a student in Berkeley's Professional Development Program started by Uri Treisman. After earning his bachelor's degree in mathematics in 1999, he went on to obtain his master's degree in mathematics from Howard University in 2001 and his PhD in mathematics from UC Davis in 2006. He joined the faculty at California State Polytechnic University, Pomona in 2007 after an appointment as a UC Presidents' Postdoctoral Scholar in the Department of Mathematics at UC Santa Barbara. He was selected as a Project NExT Fellow in 2007. Robin was a visiting professor at Georgetown University in 2014 and a Visiting Professor at Pomona College in 2017. His current research interests include both low-dimensional topology and math education.

15

When You Are Told You Can't: Do Just the Opposite

Angie Hodge

You shouldn't be a high school mathematics teacher. You should teach at an elementary school. You are a female of small stature. You would never have control of your classroom in a high school setting.

Despite the fact that I was the top mathematics student in my high school calculus class, my male mathematics teacher uttered those words to me in a manner that is still crystal clear in my mind. I went to my mathematics teacher's office to seek out advice during my senior year of high school. I was a first-generation college student who knew that I wanted to teach. However, I did not know whether I should teach mathematics, a subject I loved and was good at, or teach elementary school. I still wonder if this mathematics teacher really knew how his words would impact my entire career. Too many people do not think about the long-term effects of their words. I have made it my career mission to help others who may have been discouraged, for whatever reason, to realize their potential in mathematics and mathematics related fields.

At the time, I took those words to heart. My parents had not attended college. They were just happy that I was pursuing further education. It did not matter to them what I chose as a major. They also did not know the power of choosing mathematics as a career path. Hence, I signed up to be an elementary education major at Minnesota State University Moorhead.

During my summer orientation, we registered for our fall university courses. I was paired with an orientation counselor who was supposed to help me select these courses. She was a successful elementary education major who was near the end of her degree program. Although it is now common to take many advanced placement (AP) courses and earn many college credits during high school, back in the 1990s this was not common. My orientation counselor had trouble finding classes that I could take since I had already taken all of my English credits and a few other courses she would usually recommend to a freshman. When we reached a point where we were stuck, I asked what mathematics courses were required, since I enjoyed mathematics. My orientation counselor cringed and said those courses were hard—too hard for a freshman. She begged me not to sign up for the mathematics for elementary teachers' course, stating I was setting myself up for failure if I signed up for it.

Going against my orientation counselor's advice and signing up for the mathematics for elementary teachers' course was one of the best decisions I have ever made. In this course, I had my first ever female mathematics teacher (a female of small stature to add to it all). This instructor changed my confidence level in mathematics and in many ways changed my life. She involved me in two conference presentations, praised me for my hard work and success in mathematics, and eventually convinced me to take a university calculus course the following semester. Although I should add that she warned me that I would be in a mathematics course with engineers and mathematics majors, so I should be prepared to not be the "best" student in my calculus class—even a female mathematics teacher who saw potential in me still was not sure if I would measure up to the male engineering and mathematics students.

I signed up for the calculus course and decided to make it my mission to in fact be the "best" student in that course. I loved the calculus course and was encouraged by both my calculus teacher and my former mathematics for elementary ed teacher to change my major to secondary mathematics education, which included a bachelor of science degree in mathematics. It took me less than 24 hours to make this decision, and it started a domino effect of mathematics course taking and changed my career path. For the next

three and a half years, mathematics became my favorite pastime. I was an introvert in college and preferred going home to do my mathematics homework over going out and socializing. I was dedicated to showing people that I could do mathematics, despite my gender and body size.

Eventually this domino effect turned into my university professors convincing me to apply to and attend mathematics graduate school. By this time, I had mostly forgotten about the high school teacher who discouraged me from pursuing further mathematics. I wish I could say that the struggles ended there.

In graduate school, I faced a real analysis professor who brought me into his office to tell me this lovely advice after only one exam:

> *What makes you think you belong at Purdue? You are an American female. Purdue mathematics is not set up for American females to succeed. I also looked at your transcript file. You attended a small state school in Minnesota. You really should consider getting out now. You are not cut out to obtain a PhD, or even a master's degree, from Purdue. In the meantime, you need to change this class from a grade to pass/fail. I guarantee you will get an F in my class, if you do not change it.*

I ended up failing his course with a pass/fail. I was terrified every day in his course and my confidence had been depleted. He would not even let me see my final exam to see why I failed. I did well on the homework and felt like I also did fairly well on the final exam.

Looking back, this was a turning point for me in mathematics. I was not brave enough to contact the administration at the time to tell them my story. However, I was determined to obtain a PhD from Purdue University after this professor's behavior. I obtained very high grades my spring semester and developed a curiosity as to why I had such contrasting results with different instructors.

My interests also shifted to understanding how one teaches and how that impacts student performance. This impacted me so much so that I took a PhD-level mathematics education course the following fall semester. The instructor of this course encouraged me to apply to the PhD program in mathematics education at Purdue. I continued with my master's degree in mathematics and also continued to work and teach for the Purdue mathematics department during all of my years at Purdue, but I did switch my PhD program to mathematics education.

I dedicated my career to helping others learn their strengths rather than their weaknesses in mathematics as well as learning how people best learn

mathematics. Inquiry-based learning, or active learning, and research in mathematics education have become my passions in connection to ways to use mathematics as a pump rather than a filter (use mathematics as a way to encourage people to pursue the STEM fields rather than as a way to keep people out of the STEM fields).

The struggles are still real for me as a female mathematician. I have been asked to sit down in my own large calculus class because the students thought I was another student. I have been judged on my clothing choice in video sessions used to teach others about active learning because my skirt was "short." I have been ignored in mathematics departmental meetings. I have been in a department where I was the only female who was not married to a male in the department. The list goes on and on.

The way I persisted stems from the same words of wisdom I give to other teachers of mathematics. Find mentors. Be a supportive mentor to others. Mentor everyone. Say positive things about mathematics. Catch others doing mathematics well. Be a leader—start a program to inspire others. Help others develop a love of mathematics. Help others understand that mathematics is not just memorizing and instead that it is about critical thinking and problem solving.

Empower people with your words, your actions, and your passion for mathematics.

Angie Hodge obtained a bachelor of science degree from Minnesota State University Moorhead (MSUM) in mathematics. She went on to pursue further education at Purdue University immediately following graduation from MSUM. At Purdue University, Angie earned both a master's degree in mathematics and a PhD in mathematics education. She was a national Project NExT fellow and has since stayed actively involved in the mathematics community and the MAA. Her research and teaching passions both lie in the area of inquiry-based learning with a focus on gender equity in the STEM disciplines. Pursuing these passions, in 2012 she helped start a summer camp for underprivileged middle-school girls while at the University of Nebraska at Omaha that is focused on helping young women learn about the STEM fields through active learning experiences. She most recently took a position in the Department of Mathematics and Statistics at Northern Arizona University. There she teaches a variety of mathematics courses including mathematics content courses for elementary teachers and graduate courses for practicing mathematics teachers. No matter how busy life gets, Angie always makes time to do something active. Trail running, endurance strength training, and traveling are among her favorites.

16

Look for the Helpers

Jennifer Bowen

When I was a boy and I would see scary things in the news, my mother would say to me, 'Look for the helpers. You will always find people who are helping.'

—Fred Rogers

Courtesy of Jennifer Bowen

In October 1995, as a sophomore, at approximately 10:23 AM in the morning, after what had to be the 43rd example that semester about "blue balls in a bag," I looked around the large undergraduate lecture hall where I was taking combinatorics and graph theory and realized it: I was the only female in this course—including the professor. It had taken me seven weeks to come to this realization and the cold recognition was replaced with a deep pit in the center of my stomach. The professor was a dry lecturer; my classmates (mostly upperclass) were menacing; the community of the course was unsupportive and fractured.

This was the semester that the department of mathematics "forgot" to hire me back as a grader for calculus papers. I went in search of another on-campus job and found Learning Resources for Student Athletes. With the help of Amy, my supervisor, mentor, and superior confidante, I became a valued and beloved tutor for a multitude of Division I student-athletes studying undergraduate mathematics for the next three years—and I *valued and loved* my job.

In the spring semester, I enrolled in linear algebra; not only a pivotal course for majors, but also for my course as a major. Professor B was approachable, helpful, and supportive. He suggested summer jobs, REUs, and the Summer Mathematics Program (SMP) for Women at Carleton College. I had arrived as a real major. The following summer, of 1996, the roots of the 22-year mentoring network I rely on now were planted, with mentors Deanna Haunsperger and Steve Kennedy. I lived and "math-ed" for four transformative and pivotal weeks that shaped my future as a professional mathematician for the next 13 years.

I returned to my undergraduate institution as a re-energized junior female student, only to be reassigned to a new academic advisor, who, upon our first meeting together, after less than five minutes, surmised and decided, "You won't be able to take that Abstract Algebra course, you will probably fail it." *I got an A–*. I sought other (female) students in class for collaboration and problem solving. I spent three weeks untangling what that professor was calling (and not explaining well) a "coset"—I was victorious.

When it came time to apply to graduate school in mathematics, my undergraduate abstract algebra professor was walking to class with me, chatting about sending in applications. He was sure that I wouldn't have a problem getting in as long as I scored higher than the 80th percentile on the Math Subject GRE: I hadn't. Three months later, I was accepted at four institutions, with full funding.

In late summer of 2000, just before my third year in graduate school, I sat and wrote for my third (and final) attempt at the algebra general exam. If I didn't pass this time, I would be forced to leave graduate school with only my master's degree. My first two attempts were close, but not close enough to pass. The results for this third try were delivered to my campus mailbox in the department of mathematics, days later, on a Sunday evening. My friends and confidantes insisted on going to pick up that letter at 9 PM that night. It was emotional and cathartic in the parking lot outside: I had passed.

The following morning, I arrived in the building to prepare for the impending semester and I was suddenly stopped by Professor K, who said, "If you'd like to do a dissertation reading course with me, come see me. I like the way you work!" After the emotional evening the night before, his candor and directness was almost too much. I did, however, leave that brief conversation with renewed energy and belief in myself. Professor K sadly passed away in 2003, and his best friend and colleague, Professor F, promised him, on his deathbed, that he would help me finish my dissertation work. I was Professor F's final career PhD student (lucky #13 in his time there).

Now that I am a tenured professor, having taught, advised, and mentored at small liberal arts colleges for nearly 15 years, the opportunity for reflection is so valuable to my life's work. On a Tuesday night, in late November 2017, I had the opportunity to hear Deborah Bial, founder of the Posse Foundation, speak in Cleveland. The Posse Foundation "identifies, recruits and trains individuals with extraordinary leadership potential. Posse Scholars receive four-year full-tuition leadership scholarships from Posse's partner colleges and universities."* I am currently mentoring my institution's tenth Posse in their first years of college; the evening was transformative for me. Bial told a story about a female Fortune 500 company CEO whose advice to the organization's alumni was:

(1) work really really hard [yep!]
(2) find a mentor [helpers!] and
(3) have someone "pound the table" for you.

Mathematicians have the responsibility and charge, in 2018 and beyond, to pound the table for the struggle of underrepresented groups in mathematics: *it is real*.

Jen Bowen earned her PhD in mathematics at The University of Virginia in 2005. She has taught at The College of Wooster since 2007. She is presently an associate professor of mathematics and chairperson of the Department of Mathematics and Computer Science at The College of Wooster. Her interests include nonassociative algebra, undergraduate knot theory, and advising and mentoring of underrepresented populations in mathematics. Jen hopes that her beloved mathematical daughter, Carys, won't need to write a similar 'struggle' piece in 35 years.

* www.possefoundation.org

17

Good, But Not on the Team

Tim Chartier

Courtesy of Davidson College

I grew up playing a lot of sports in my town. We played baseball in a parking lot with a tennis ball using a selection of bats and tennis rackets for hitting. We played basketball on an asphalt court that only went a bit beyond the free throw line with a basket stretching about nine feet, rather than ten, off the ground. My basketball game was honed for those courts with their chain-link nets. I had a personal playbook full of dunks, layups of various forms, and trick shots. Yet, if you asked me if I was a basketball player, I'd hesitate. I knew I couldn't make a team since their games were played on a full court with regulation equipment.

I also grew up performing puppetry and, later, mime. In junior high, my book reports were a class favorite as they were puppet shows interspersed with humorous commercials to maintain attention in the overall production. Before graduating high school, I was performing throughout the Philadelphia area and teaching and presenting in national and international settings during college. I was a puppeteer and mime.

Mathematics also had a place in my life amidst all the made and missed shots and hours in the basement practicing puppetry and mime. On long car trips as a child, my sister would play word games. I'd sit beside her in the back seat playing number games—counting things, looking for patterns in license plates, and coming up with other numeric ways to pass the time. Throughout my elementary and secondary schooling, math was the easiest class for me. I found the problems interesting but, overall, they were challenges to complete so I could move onto honing my performing skills or getting outside to play sports. If you asked me what I enjoyed, I'd talk your ear off about theater or sports. I thought of math as simply something I was good at.

Fast forward to the me who pens this reflection, and I can easily say that I'm a mathematician. Yet, it occurs to me that it took considerable time for me to recognize this as being a part of my identity. I was good at math but not actually a mathematician. My identity lay largely in my art and athletics.

An important aspect of why I didn't see myself as a mathematician also lay at the foundation of why I wasn't sure how to respond regarding my skills in basketball. I played basketball, but not in the traditional sense of playing on a team. Sometimes, I played with as many as 20 children in my town. We'd change the rules to keep the game entertaining and so no one sat the bench. I was good at playing variations of traditional basketball, which seemed to distance me from being an actual basketball player.

As an artist, I performed in traditional ways. I visited churches and libraries throughout the area and filled meeting spaces with laughter as I presented to groups of varying sizes, even in the hundreds. I saw myself as a puppeteer and mime.

Math? I didn't join the math club. I didn't take part in competitions. In fact, what I didn't do seemed like a longer list than what I did. So, what about math? I could do it, but I was clearly not cut out to be a mathematician. Similarly, I played basketball but would never make a team. If math had a team, I wouldn't have made that team either.

There was another layer, though. A number of my friends in both performing arts and sports didn't like math. They struggled with it and frequently flaunted a certain level of number sense by treating "math" as a four-letter word. In my mathematics classes, it seemed as if math, sports, and the arts were completely disconnected. My interests seemed necessarily disparate and disjoint, however connected they were within me.

Was I a mathematician? Yes. However, I needed time to see myself in the field. The first step happened in 11th grade. I was quite ill and taking

all my classes at home. A college professor and family friend, Marvin Rolfs from Fort Hayes State University, was visiting family and offered to tutor me. Early in our time, Marvin noted that, while I was very good at memorization, we should work on deriving and understanding as many formulas as possible. That experience changed my view of mathematics. I worked to remember as little as possible while still able to accomplish the required tasks. It was almost as if Marvin gave me the fundamental Lego pieces of my mathematical content. My job was to build with them. I marveled at the structures we created. I'd sometimes sit in awe while Marvin responded with his warm, gentle chuckle. I don't believe I saw myself as a mathematician at this point. But, I began to enjoy it.

The next big step occurred years later in a college proofs class, semesters after completing calculus. That class introduced me to mathematical elegance. Proofs had artistry and creativity. I talked to my friends about proofs. I would wake up at night, still with performing arts ideas, but now with new notions of how to prove a result. Indeed, I was a mathematician.

Now, I'm a mathematics professor at Davidson College. Interestingly, I created a mime show that introduces mathematical ideas. I have performed it around the world with my wife, who also performs mime. My mathematical research focuses on sports analytics, enabling me to work with teams in the NBA, NFL, and NASCAR. In the end, these interests—which I once saw as disparate—interweave in my career as they have within me since childhood.

So, is there a light at the end of that struggle, that struggle that's real? There was for me. But, again, to know me fully in the field, it's important to know that echoes reverberate. I can still feel apart from the field in which I have spent years learning and teaching. I'm an applied mathematician who is a professionally trained mime and works in sports analytics. In life's more challenging moments, my path can seem to have diverged from the mathematical into possibly the more motivational and creative. When I persist and move through such uncertainty, I stand firm and note that I am, indeed, a mathematician—simply a nontraditional one—at least for now. Excitingly, the field never stops evolving.

Tim Chartier is a professor of mathematics and computer science at Davidson College. He frequently works on data analytics projects with professional teams in the NBA, NFL, and NASCAR. He has served as a resource for a variety of media inquiries, including appearances with Bloomberg TV, NPR, the CBS Evening News, USA Today,

and the New York Times. *With a group of over 50 student researchers, Tim Chartier supplies analytics to Davidson College sports teams. He was the first chair of the Advisory Council for the National Museum of Mathematics and worked with Google and Pixar on their K–12 educational initiatives. As an artist, Tim, along with his wife Tanya, has trained with Marcel Marceau. They've performed their mime and math presentation across the United States and in Japan, South Korea, the Netherlands, and Panama. The use of their art in this way has also been covered by NPR (indeed, a radio interview about mime) and the* New York Times.

18

Othering and Such Climatic Joy Killers

Arlie Petters

I remember being giddy with excitement to attend the welcoming reception for my entering class of math graduate students. I walked into the room and heads turned towards me. Feeling out of place, I walked over to two student-looking faces. One happened to be a fourth-year graduate math student and the other was a first-year like me. I introduced myself and, because I wanted a quick exit, I asked the more senior student how to get to the main math office. He told me that when I walked out the door, I should make a left, walk down the hallway, make another left, and it would be on my right. "Or, you could tie a rope to the ceiling and swing over to the other side," he said with a mischievous grin. The first-year student turned red with embarrassment. It did not matter whether the senior student thought of me as a monkey in a tree, Tarzan, or something else; his decision to engage in an unnecessary framing that could provoke a negative stereotype was telling. I

This story is excerpted from "Belonging" in *Notices of the American Mathematical Society*, February 2018, pp. 120–123.

quickly responded, "I see that you're going to be an asshole," and I walked out of the room. Here I was, looking forward to being part of a new community of mathematicians and then being made to feel unwelcome at the onset.

I went directly to my apartment and started packing. My mind was racing and I was angry: "To hell with them. They turned around looking at me because I am a person of color. I am leaving this place. To hell with these people." As I started calming down, a counterintuitive thought occurred to me: "What if the others in the room weren't like him? What if they turned around and looked at me because they don't often see someone like me in an entering class and were curious to get to know me? … If I leave, this guy will win. I refuse to let him win." My psychological bounce back was that he had brought the fight to me, and I refused to cower in fear or run away in anger. I had briefly allowed him to hijack and taint my perspective. And, even worse, by allowing him to make me angry, I had given him power over me in that moment. Never again. The emotional-intelligence battle was on. Would I have had such a fight-back spirit in the academic sphere if from pre-kindergarten my sense of self had been chipped away, bit by bit, by individual and institutional racism? I doubt it. Fortunately, I was raised until the age of 15 in Belize by a loving and resilient grandmother who strengthened me internally, fortifying my identity and allowing me to maintain its structural integrity in the face of undermining forces.

I was not naive about the epiphany that caused me to stay. My hypothesis that most people in the room were not like him needed to be tested. But I had enough internal energy and grit to hold on to it by blind faith in the short term. The energy sustained me through the long hours of hard work needed to perform very well on my homework sets. And the grit enabled me to bear the anxiety that maybe most people in the environment did not really care for my being there and did not think much of me intellectually. In my case, I was fortunate to discover with time that most of the people were not like that graduate student. I had a perceptive and supportive thesis advisor and a positive interaction with the majority of the other math and physics graduate students and faculty. That young man had acted as if he owned the place. To me, he had a warped sense of belonging and entitlement that made him feel confident enough to treat me in a demeaning way without consequences.

I wish I could tell you that my experience was an anomaly. Over the years I have mentored a host of underrepresented minority students and listened to their experiences. They range from regular racial micro-aggression, through "oppressive othering," to more overt examples, like being the only

one not invited to a bus outing organized by fellow math graduate students. A sense of belonging involves one's personal belief that one is an accepted member of an academic community whose presence and contributions are valued. This is important not only for the mathematics community but also for education and our society at large. At the convocation for Duke's entering 2017 undergraduate class, Stephen Nowicki emphasized to our students:

> *We only learn best from each other and teach each other well if we all feel like we belong. We can only achieve the excellence that lies in the potential of the different people and perspectives, the different aspirations and ideas we've brought together at Duke, if everyone feels equally that Duke belongs to them.*
>
> *There's another important thing to understand about what it means to belong, which is that 'belonging' does not mean 'conforming.' … The excellence of this place emerges from the very different kinds of people who join our community. To diminish those differences through conformity would only diminish our excellence.*

If we truly believe that diversity in all its dimensions is a key driver of excellence in our educational institutions and increases the probability of intellectual breakthroughs, then we cannot ignore the implicit biases directed toward underrepresented minorities and women. Actionable first steps a department can take as part of fostering a welcoming culture are to assign thoughtfully chosen mentors to incoming students and faculty; to advocate inclusion, acceptance, and understanding; and to promote effective ways to engage diversity. Imagine for a moment that you are a newcomer. Having someone in your department teach you the ropes and advise you from their own experiences is part of an onboarding that tells you from the beginning that you matter. Usually it is through such a relationship that your trust in the environment grows. By trust, I mean that you can allow yourself to be intellectually vulnerable without fear that your admission of the need for help or clarity will be attached to your race, ethnicity, gender identity, or social-class history. For example, you can feel secure enough to admit that you have certain gaps in your math background and allow the mentor to assist you with filling them. And you can ask faculty and seminar speakers questions about mathematical issues that are unclear to you.

For many underrepresented minorities and women, the issue of belonging in mathematics has been a continued fundamental challenge. I believe that an integral part of keeping our field vibrant and relevant is for its participants to welcome everyone, knowing that anyone can get better at mathematics through an ample commitment of time and energy by teacher

and student. Equally important, one should not only be welcoming at the door but also give people a chance to add value inside. Belonging is indeed a foundational human need, which when nurtured can bring out the best in all of us, enabling our community to maximize its excellence. In the end, mathematicians are the custodians of mathematics. The onus is on us.

Arlie Petters is the Dean of Academic Affairs for Trinity College of Arts and Sciences and Associate Vice Provost for Undergraduate Education at Duke University. He is also the Benjamin Powell Professor of Mathematics and a professor of physics and economics. He has written five mathematical books, including a monograph on gravitational lensing and a textbook on mathematical finance.

19

Black, Female, … Bigger

Candice Price

As a child, I didn't know what I really wanted to be when I grew up. I mean I had some grandiose ideas like being the first black woman president; but, I also had much less lofty goals like working in a movie theatre. I loved watching movies, and I loved movie popcorn. As a child I just assumed if you worked at a movie theatre, you could have as much of each as you would like. The thing is that there wasn't anyone telling me anything different. I mean no one was telling me anything at all really about what I could be. My father was a nurse who worked third shift. He was always tired and never liked talking about his job. And of course as a black male, the nurses I saw on TV didn't look like him, so I never really understood what he did. I couldn't even guess. My mom was an adult-education teacher. She taught classes to people studying to be medical assistants. She worked long teacher hours, and although I often visited her classroom after school or on weekends, we never really talked about her profession either. Plus, I had no black teachers growing up, so I didn't really understand what my mother's job was like either. I didn't see reflections of my parents in their respective professions.

Now that I am a professor, I understand my mother's job so much more. I understand the late hours of grading or lesson planning. The conferences

to hone her craft. The interactions with students outside of the classroom, maybe at the grocery store or the mall. But still, no one talked to me about teaching or being a teacher, let alone teaching math! And as I look back, it's strange that no one talked to me about it since I was involved in so many STEM activities as a child: the Math, Engineering, Science Achievement Program, math competitions, summer STEM camps, and a computer animation internship. Yet, no one ever spoke to me about my mathematics potential. The only time I can recall career goals for me being discussed was once during a conversation with some high school friends. They were discussing their goals of starting a school. They decided that one of them would be the teacher, the other would be the principal and that I could be the "yard duty." The "yard duty" was basically K–12 campus security. I now understand something about that exchange: My friends did not see my potential. They only saw my size and my race. This is something that has always been a struggle for me and a reason why I think no one discussed career goals with me. I am rarely viewed as more than race and size. People rarely see me as kind, funny, intelligent, thoughtful and driven. I want so much more than to be defined by just my race and my size.

When I did figure out what I wanted to pursue as a career, motivated by the lack of black female professors I had in my life, I started my PhD journey. Yet, even though I knew my career goal, I was not protected from people defining me by just my race and size. Upon meeting the chair at the time, he mentioned he confused me with another student as we are both "black, female, … bigger". A fellow graduate student later that year also asked me, "Which one are you? I know there are two of you." The struggle of navigating a space where you are visible (size and race) and also invisible (one of the black students) when all you want to do is prepare yourself for a career you never even knew you could possibly have is debilitating. It creates the kind of fatigue that can only be understood and remedied by others who are forced to navigate spaces in a similar way. But it also makes you think in the back of your mind: Am I in the right career? Because the struggle of constantly proving you should be in that career, in that hospital, teaching that class, at that research conference while the general profession does not reflect you, is very real.

Candice Price was born in Long Beach, CA but raised in Sacramento. She earned her bachelor's in mathematics from California State University, Chico, her master's in mathematics from San Francisco State University, and her doctorate from the Univer-

sity of Iowa. Currently, Candice is an assistant professor in the Department of Mathematics at the University of San Diego. Her research area is primarily in the area of DNA topology but she is currently working in various areas of mathematical modeling including using social network theory to study evolutionary success and issues around gerrymandering. Her service mission statement is to create and contribute to programs that broaden the participation of underrepresented groups by focusing on strong mentoring and research networks.

20

Moving Beyond Affirmative Action for Men

Alice Silverberg

Photo Courtesy of The Ohio State University

I joke that all of my higher education was at single-sex universities … but unfortunately for a sex of which I'm not a member. The disparities between the way men and women were treated at those universities ranged from serious to laughable.

I grew up going to New York City public schools when they were unsafe and the city was verging on bankruptcy. Neither of my parents graduated from college, so my siblings and I had to learn for ourselves how to cope with the Ivy League and Seven Sisters colleges.

Harvard College didn't admit women to the Class of 1979 (even though that's the college I went to). The female students were all admitted by Radcliffe College. The Harvard/Radcliffe ratio at that time was 2.5 to 1 by fiat, having gotten there gradually from a ratio fixed at 4 to 1 a few years earlier. People have told me that affirmative action must have helped me get into Harvard. They don't realize that Harvard's affirmative action favored men and not women; discrimination against women was institutionalized.

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Our value was conveyed to us in trivial ways before we even started. With the acceptance letter, Harvard sent the boys a postage-paid envelope for their response, and a fancy certificate suitable for framing stating that they got into Harvard. The girls needed to put stamps on their response envelopes. While Princeton alumni joke that Princeton's Latin motto translates to "God went to Princeton," I joke that Radcliffe's Latin motto translates to "Radcliffe has no money."

The female students were known as "Radcliffe bitches." Some professors made it clear that they expected the men to go on to top graduate schools, while they expected the women to teach high school.

Harvard declared itself coeducational in 1999 [sic], but its 360-year legacy has a lingering impact, for example in faculty gender ratios. There were no female tenured mathematics professors in the Harvard, Cambridge, or Princeton math departments when I was a student there, or any time before. Neither Harvard nor Cambridge had any in the 20th century, and Princeton's first was in 1994.

Churchill College prides itself on being "the first of the formerly all-male Cambridge colleges to vote to admit women." It is less proud that it was the last Cambridge College to be founded as men-only (contrary to Sir Winston's wishes). Churchill College first admitted female undergraduates in 1972. The Master while I studied there in 1979–80 had voted against admitting women.

That women weren't welcome at Princeton, which went coeducational less than a decade before I applied to graduate school, was signaled in ways ranging from blatant to subtle. A group calling itself the "Concerned Alumni of Princeton" agitated to revoke coeducation. The ratio of men's rooms to women's rooms in the math building was 3.5 to 1. Some time after I graduated, someone who has been an AMS President told me that the reason there were no female students when he went to Princeton was that none were good enough to be admitted; he wasn't aware that women were barred.

I learned many things from my experiences. I learned that decisions that should be based on merit and fairness are often (subconsciously) instead based on empathy. This unfortunately leads to people favoring people who remind them of themselves, and people finding it hard to believe that those isomorphic to themselves can do bad things.

Something that helped me survive as a mathematician was that I'd rather listen than talk. Putting yourself in someone else's shoes is a useful game. Sitting around after dinner with other students (i.e., procrastinating instead of studying), we tried to figure out why we disagreed on something that

seemed obvious to each of us. We could usually trace our differences to our own experiences or our family's values (and our acceptance or rejection of them). This helped us to see that there can be more than one valid viewpoint. One learns more by listening than by speaking.

I try to have a sense of humor and to be bemused rather than angry or resentful (though I don't necessarily succeed at that), and I try to remember what's important and not get stressed about things that aren't. I try to view the world with a sense of adventure and an appreciation for the absurd. (It helps that I haven't yet figured out that I'm not Alice in Wonderland.) Mathematicians (and perhaps people in general) are a lot like children, with both the good and the bad that accompanies that. I'm eternally optimistic that, like children, they (we) have the capacity to learn and become better.

The mathematical community is a lot like a family. It's a collection of people, some difficult, some complicated, but to a large extent we care about each other.

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Alice Silverberg is a professor of mathematics and computer science at the University of California, Irvine. Her research areas are cryptography and number theory. She received her undergraduate degree summa cum laude from Harvard University, a master's of advanced studies degree from the University of Cambridge, and a master's degree and PhD from Princeton University. Before joining UCI she was a professor of mathematics at the Ohio State University. Alice has been awarded Humboldt, Sloan, IBM, Bunting, and National Science Foundation Fellowships, and she has held visiting positions at industrial labs and international research centers. She consulted for the TV show NUMB3RS *and the documentary* Julia Robinson and Hilbert's Tenth Problem*, and occasionally writes mathematically-inspired Scottish country dances.*

21

Struggling with the Messaging of Mathematics

Rachel Weir

Courtesy of Allegheny College

Throughout our lives, we absorb messages about mathematics. Everyone's understanding of and reactions to these messages is different but, looking back, I see the many ways in which I have been affected by them and I also see the negative effect that they have on my students and on society in general. My instinctual response was to adapt and conform and it took me many years to realize the toll that this took on me and to begin to push back.

As a child, I excelled academically and, to my siblings' chagrin, I was always labeled "the smart one." I was dux (valedictorian) of my high school and finished as the top female student in the national exams in New Zealand that year. Mathematics was always the primary measure of my "smartness," starting in elementary school and continuing into my college years, and I generally took mathematics courses with the grade level above mine. Here the message was that, because of my mathematical aptitude, I was "special," I was a "math person," I was "smart," and it became my mission in school to hold on to these labels.

In graduate school, I started to struggle with mathematics for the first time. Suddenly the mathematics was more abstract than anything I had ever encountered before and I wasn't sure how to adapt to this. Given how tied

my identity was to my mathematical abilities, this was devastating to me, leading me to doubt my abilities and my place in the mathematical community. I came close to dropping out but in the end I stayed, if only because I couldn't see myself doing anything else.

Life became a little easier once I got past the courses and the qualifying exams and started my thesis research. I discovered the joy of immersing myself in mathematics, reading and rereading an article until the results became familiar friends, and then developing my own mathematics, being fully absorbed by the revelations, the disappointments, the frustrations, and the amazing successes. But at the same time, there was always fear. The crippling fear that made it difficult to begin a research project because any failure would suggest that I wasn't cut out to be a mathematician, that I wasn't as "smart" as everyone had thought.

And then, on top of this, the broader fear that I didn't belong. Through my undergraduate and graduate years, you could have counted on one hand the courses I took that were taught by tenured or tenure-track women. A graduate school course on several complex variables, taught by a male visiting scholar of great renown, was the only course that I ever dropped, after it became clear that he felt that women were not capable of understanding the mathematics he was discussing (the only other woman in the course also withdrew from the course). In addition, there were few women among the graduate students and many of them were there "just" for a master's degree, so they could go on to teach.

This message was also conveyed implicitly at conferences. The women who gave the plenary talks in my area stood out as glimmers of hope, but there just weren't enough of them to counteract my feelings of being on the outside. Add to this that I was a gay woman in a space where it felt that sharing anything personal would dilute your credibility as a mathematician. My response was to remain closeted for a long time and to distance myself from my research community once I realized that coming out was essential to my mental health.

And then there were the messages about teaching mathematics. As an international student, I was required to teach every semester as part of my graduate student assistantship. American students qualified for research assistantships and typically didn't have to teach as much and, at the time, it felt as though we were the underclass. I eventually became a Graduate Student Mentor, helping to train other graduate students and postdocs, but I learned that if you wanted to be taken seriously as a student of mathematics, you needed to focus on your research and to take care of your teaching obliga-

tions as quickly as possible. This continued into my postdoc, during which the only person who ever observed my classes was my collaborator, after I invited her to visit so that she could mention my teaching in the letter of recommendation she was writing for me.

On top of this, although as a graduate student I taught calculus using cooperative learning techniques (the particular approach we used was pioneered at my graduate institution and continues today), those approaches didn't penetrate beyond those courses. Every mathematics course that I took as a student was lecture-based and, for a long time, every course I taught was as well. And the messages that I received during my tenure-track years led me to understand that I needed to stick with the traditional methods or risk losing out on tenure.

I didn't truly reflect on any of this until, as a tenured faculty member at a liberal arts college, I started to think deeply about how I could improve my students' learning experiences and I learned about things like growth mindset, productive failure, active learning, stereotype threat, unconscious bias, weed-out courses, and sense of belonging. I started to see my own experiences in the stories I read and I learned that the points in my career when I felt like a failure were part of a broader narrative. This knowledge has emboldened me to begin to speak up, for myself and for my students, and has pushed my career in new and exciting directions. The messages that I absorbed shaped my professional self and, while I have no regrets about the path that I took, I do wonder how things might have been different if mathematics as a discipline was more inclusive, more diverse, more humble.

Rachel Weir is a professor of mathematics at Allegheny College in Meadville, PA, where she lives with her partner and daughter. Originally from New Zealand, she earned her BSc(Hons) in mathematics from the University of Auckland in 1996 before moving to the U.S. for graduate school. She earned a PhD in mathematics from the University of Michigan in 2001 and was a Whyburn Research Instructor at the University of Virginia from 2001 until 2004. Her mathematical area of interest is function-theoretic operator theory but, in recent years, her focus has shifted to improving the success of students in college mathematics courses through the use of non-traditional approaches such as flipped learning, mastery grading, and inquiry-based learning. Rachel spent part of her 2017 sabbatical learning to surf in New Zealand and hopes to be surfing the Great Lakes soon.

22

The Harassment Is Real

Pamela Harris

Courtesy of Cesar E. Silva

"Where do I turn in your teacher evaluation form?" followed by, "Will you now take my late homework?" This was said to me one week before the end of the semester.

This is a quote from a white male student, and it is not unique. To label interactions like these as hostile would not get to the root of how the words continue to affect me. Why would students feel inclined to treat their professor this way? How does my being a woman of color negatively shape my students' interactions with me?

I may never know the answer to these questions and, in fact, I have stopped trying to answer them. Sadly, these experiences have been common throughout my journey to becoming a mathematician. They are the valleys where I have hit rock bottom and from which I had to climb out in hopes of finding a place where I belong, where my uniqueness is valued, and where my worth is not questioned. While I have persevered, I have not found this place, and it may be that such a place doesn't exist—it is the unicorn of academic institutions.

I cannot pinpoint the first time I began to feel like such an outsider. Interactions with students who question my authority or knowledge or try to manipulate me certainly contribute to this feeling. But sensing that I don't belong is something that stems back to graduate school, when I discovered that I was suffering from impostor syndrome. I vividly recall that my advisor would ask me questions, and I would meekly begin every answer with "I think …". He would kindly point out that I knew exactly the definition and what I should be doing, but the way I responded did not match what I knew. I still do this—responding to questions in a way that does not fully own my knowledge. It has taken me a long time to discover why I do this, and part of it has to do with the fact that my family, as is traditional in Mexican culture, highly values humility.

So, how can I remain humble while internalizing my own knowledge and sharing it with an academic community in which people thrive based on how much they know and how eloquently they can explain it? These concepts seem to contradict one another. Because of this tension, I may never fully overcome impostor syndrome. But maybe the goal doesn't need to be overcoming it. Instead, as Dr. Ana Mari Cauce, President of University of Washington, stated at the 2015 Latinxs in Mathematics conference, "The impostor syndrome has never stopped … but it also has never stopped me."

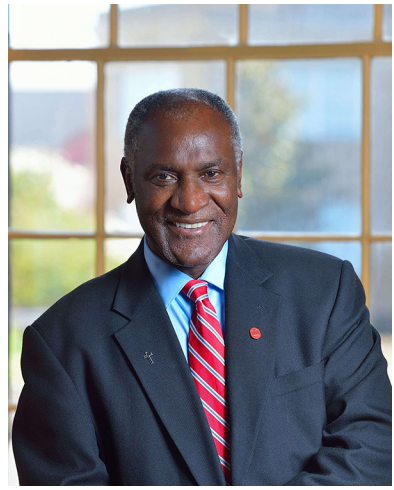
Although academia was not built with people like me (e.g., women, immigrants, people of color) in mind, I will continue to help build communities of peers and mentors who help each other feel like we belong and who value what makes each of us unique. We need to foster communities where we are all worthy of having productive and successful careers as mathematicians.

Pamela Harris is a Mexican-American assistant professor in the Department of Mathematics and Statistics at Williams College. She received her BS from Marquette University, MS and PhD in mathematics from the University of Wisconsin-Milwaukee, and was a Davies Fellow of the National Research Council with a dual appointment at the United States Military Academy and the Army Research Lab. Her research interests are in algebra and combinatorics, particularly as these subjects relate to the representation theory of Lie algebras. In 2018, she was selected as the Outstanding Mentor of the Year by the Davies Center at Williams College and she was one of 50 women featured in the book Power in Numbers: The Rebel Women of Mathematics.

23

Sustaining Through Mathematics

Donald Cole



"Shhh," my mother cried in a stern tone not characteristic of her normal demeanor. Even a sixth-grader like me could tell that something of significance was being broadcast on the television screen. "The University of Mississippi (UM) is being integrated!" she said in a serious tone. I looked at the television only to see a mob filled with mean, angry, vengeful faces distraught at the idea of being invaded and forced to racially integrate what had been a proud and closed society. I recall saying to myself, "I would NEVER go to a school like that!" Fast forward just six years. I find myself entering "that school" as a freshman with pockets of resistance still present and just as prominent in their defiance; encountering professors who had vowed to never teach someone like myself, staff members sworn to never serve an individual like myself, and students who came there just to rid themselves of people like me. It was an unpleasant experience and by my sophomore year, I had taken on much social and physical abuse, been explicitly and repeatedly told that I could not major in mathematics, and even expelled from the university for participation in racial protests on campus.

I recall how mathematics "proved" to be my therapy during those turbulent times. As the only person of color in uncomfortable and often hostile

classes, maybe I couldn't outsmart 'em, but surely I could outstudy 'em and be assured of passing grades as well as earning their academic respect—even if it were not directly expressed. Before expulsion that's exactly what happened through several mathematics classes including the calculus and differential equations series.

After transferring and entering other institutions to earn my bachelor's and master's degrees, I returned to the UM, yes, the school that had expelled me, to pursue a PhD in mathematics. Unsure if I had made a wise or foolish decision, uncertain if it was the nature of the degree, complexity of the material, or a lingering disdain toward my presence—the pursuit proved to be a painfully difficult journey. This journey twisted through a perplexing path of uncertainty and self-doubt compounded by the presence of voices accentuating my weaknesses, with the only encouragement coming from those who had absolutely no influence on the process. I found myself resorting to mathematics itself as a mechanism to mask my social discomfort and gain much needed self-respect. Indeed, the bittersweet taste of that journey lingers forever. Bitter were the accidental oversights of invitations to outside departmental gatherings, but, "Ha-ha I finally conquered you Mr. Measure Theory!" added the needed nectar to sweeten that oversight. Bitter was the taste of having insufficient help to cross the wide treacherous "Ms. Algebra" river known for ending doctoral dreams, but sweet was the taste of recognizing enough of that Galois theory on the comprehensive exams. Even today, the "bitter-sweetness" of that mathematical journey avails much to the analysis required in the decision making of my current academic administrative life. I make many complex administrative decisions and without conscious realization: I dissect their structure, analyze those substructures, find multiple resolutions, eliminate the unpractical ones, regroup those structures and set out to prove that my decision formulates a good, if not optimal, solution. More simply stated, I use my mathematical reasoning to solve those non-mathematical problems.

The end of my doctoral journey clearly revealed two facts (actually three): I had completed my PhD in mathematics, and I was ready to tackle the world. (The third is of a religious nature and is reserved for a separate article!)

The art in mathematics had sustained me through the science of mathematics. Following an enjoyable and successful career in industry, I transitioned to academia at Florida A&M University. I couldn't imagine being "paid" to philosophize in an analysis or algebra class. "Let me tell you what the author really means" turned out to be one of my favorite lines, and it

still is! Nature doesn't give up her secrets easily, and mathematical research is difficult—even for the best. I didn't realize it, but I needed a good mentor at that time. Research success came slowly, and resorting back to a difficult theorem I had mastered or topics I thoroughly understood kept me going and provided relaxation, reassurance, self-assurance and even pure joy. While living the life that many others dreamed about, my world was uprooted by a phone call inviting me to return to the University of Mississippi as a faculty member in the department of mathematics and as a university administrator. Should I even seriously consider this offer? Had they forgotten that I was a "white-sheep" once expelled? Encouraged by my wife, we did the MLK thing: "took the first step without seeing the entire staircase." A 25-year glance back on that plunge reveals a remarkable view of not only life, but also the life of a nation, and that of our mathematical discipline. Who would have ever thought that an institution submerged in a pit of racism would today be cited as "too liberal" in its thinking? Or a native son of that institution once deemed an outcast would someday become a member of the president's cabinet? Or that Fermat's Last Theorem would fall to an eloquent but elaborate proof? I have faith that the next 25 years will bring forth equally exciting results: for me, for our nation, for our mathematical discipline, and for you!

Donald Cole is an assistant provost and associate professor of mathematics at the University of Mississippi, where he plays an active leadership role in policy making, teaching, research, and diversification. He attended high school in Jackson, Mississippi, and holds a PhD in mathematics from the University of Mississippi, MA degrees from both the State University of New York and the University of Michigan, and a bachelor's degree from Tougaloo College. Donald enjoyed a successful career in the aerospace industry with The General Dynamics Corporation – Fort Worth Division (now Martin Marietta) before moving into academia at Florida A&M University. He joined the University of Mississippi in 1993 as the Assistant Dean of the Graduate School and an associate professor of mathematics. He has also served as Assistant to the Chancellor for Multicultural Affairs and Associate Dean of the Graduate School. As Director of the Ronald McNair Program and the LSAMP programs, more than 60 minority PhDs in a variety of disciplines are attributed to his grants, teaching, and mentorship. He is known around campus for both his brisk walking pace and his elaborate necktie collection. In his spare time, Donald enjoys photography and aquariums.

III

Can I Really Do This? How Do I Muster Through?

When looking back, many moments in which we all struggle end up being isolated moments in time, but when we're *in* those moments, it can seem like an eternity! It is hard to *know* deep down that it *is* possible to get through those moments, and sometimes the best advice any of us can give others is to talk to someone else, whether it be a family member, a trusted friend, the professor for your class or just one you've formed a bond with, or someone who's been through a similar struggle in the past.

24

Good Things Come to Those Who Shower

Robert Allen

Courtesy of University of Wisconsin - La Crosse

One quality that mathematicians possess is a propensity to persevere (A for alliteration). However, this is not always internalized; there will always be times when we doubt that our efforts will yield results. For me, this was never so apparent than during my last full year of graduate school.

In my final year of graduate school, many things were racing around in my head. But the one thing that was always on my mind was my dissertation. During the writing phase, I was never more than five feet from some draft of a chapter or scrap of paper with crazed scribblings. Close to the end of this process, panic set in. The single worst realization hit me. I've pushed everything I can't do until the end!

For me, this happened when I was trying to devise a proof for the compactness of a certain operator acting on a very general space of functions in several complex variables. For weeks, I struggled. The struggle was real… and complex! I recall writing down the same definition over and over, in many equivalent forms. Searching the literature, I found various techniques and test functions that I could not apply. Hope was fading with every passing day. Meetings with my advisor yielded the same result: "Keep thinking about it." Although great advice, it was not what I wanted to hear.

At this point, my brain was firing on all cylinders. This was causing me to lay awake at night thinking about how this result was going to keep me from earning my PhD. In times like these, a long hot shower usually relaxes me enough to fall asleep. Let me set the scene in a non-creepy way. My bathroom is a small square room with a shower stall with a door (which will become important momentarily), a sink with mirror, a toilet, and a rack for storage (which are not important to the story). As I am standing under the scalding water trying to wash away the anxiety and frustration of my situation, a statement/question pops in my head. "I *must* know something about compact operators. What do I know about compact operators?" The answer was three words: The Spectral Theorem. That's right, the glory of every functional analysis class on the planet.

By this time, the steam had covered the shower door, creating a wonderful writing surface. So, I started writing the Spectral Theorem. I then feverishly began to connect the dots, and, in the fog of a steaming shower, I wrote the cutest result in my dissertation. I immediately jumped out of the shower to grab a piece of paper and a pencil to jot this idea down. Yes, I was running around my apartment in my birthday suit, dripping wet, giggling like a preteen watching *Twilight* for the 18th time (Team Jacob).

Once the adrenaline worked its way through my system and I had put some clothes on, I sat down to write the results and send them to my advisor, proclaiming victory! Then all the self-doubt flooded back in the single thought: "What if there is something wrong with my proof that I can't see?" As we all know, plenty of mathematics that is "proven" at odd hours of the night turns out to be not true. So now I'm lying in bed not sleeping, not because I cannot prove this result, but because I'm afraid the proof is wrong. Fortunately, this story has a happy ending. The proof was correct. To this day, I look at that five-line proof in my dissertation as the best math I've ever written.

So, the moral of this story is to "keep thinking about it" … and shower every day.

Robert Allen is a professor of mathematics at the University of Wisconsin-La Crosse. He received his PhD in Mathematics from George Mason University in 2009 under the supervision of Flavia Colonna. His research area is operator theory, and specifically he studies weighted composition operators on Banach spaces of holomorphic functions. When he is not teaching or staring at a blank sheet of paper trying to prove a theorem, you can find him trying to learn to play the piano, baking something he has seen on The Great British Baking Show*, or playing video games.*

25

Winning by Impression

Robin Blankenship

The class was algebraic topology and I found the material to be fascinating and fun. I had a strong group of study buddies with whom I worked regularly on the homework. We also had several in-class tests. To my great surprise, my male peers would receive A's on their assignments and tests, while my scores were about 30% every time.

In the beginning, I compared my work with my peers to try to figure out why we had such wildly different scores. I found no major deviations from process, notation, and deductive reasoning. I approached the professor as politely as I could, telling him that I must have a major gap in my understanding, because I was not able to find a flaw in my reasoning or proof structure. The professor told me that in fact my proofs were correct; however, "On the one hand, I grade what you write down. On the other hand, I grade by impression, and I just have the impression that you do not know what you are doing." I left immediately with consternation. How can that argument be countered?

Next, I took my work to other topologists in the department to ask them to give me advice for improvement. I just wanted to make sure I was learning the material. They confirmed each set of solutions were essentially correct

and had no idea why he was failing me. I then took all of this to the department chair hoping for intervention. I don't remember exactly what the department chair told me, but absolutely nothing changed following that meeting.

I was on my own for this particular challenge; I devised a plan. I utilized the library to access a handful of papers my professor had published in the last few years. I secretly devoted myself to the articles, learning as much as I could about each one. I sped through our textbook to develop a sequence of questions about our future material, phrased in such a way that one of his papers specifically addressed the answer.

I began to implement my plan immediately. I came to his office to ask the questions twice per week. Each visit was carefully planned so that I knew I had a class to attend within 10–12 minutes of my arrival. "Hi there, do you have a couple of minutes? I was looking at the next section that we are going to cover, and I think I'm pretty good on some of it, but there's this one thing on page 157 that has me thinking. Does this mean … (insert question that his research answers here)."

Oh my goodness, he would become so excited. His eyes would light up. He began to passionately talk about the material. Coincidentally, he had written something about this just a few years ago! Then, right as he was getting fully into the explanation, I'd sadly tap my watch on my wrist and apologize profusely. "I'm so sorry, but I have to go to class now. Thanks so much for getting me started; maybe we can talk more about it later. Thanks again for your time! This is really interesting!"

The end of the semester came, and we had a final presentation to give to the class on a topic of our own choosing. This time I visited his office to ask him for advice on a potential topic. He was anticipating my visit, and thought I might like to read some articles he had written. I smiled and thanked him as he handed me the same set of papers that I had been studying for a month. "Oh my goodness, look at the time. I'll check them out later!"

I brought the papers back to him the next week. Apologetically, I explained that they were just a little too advanced for me, kind of boring and with so much notation everywhere. I wanted to give a really interesting presentation that I thought the class would enjoy. I pulled a genus-three coffee cup out of my bag, a recent gift from a different professor, and thumped it on his desk. "I'm going to do my presentation on this coffee cup instead!" The look on my professor's face was astonishment. His mouth opened, but no words came out. I tried my best to look sad and slightly ashamed, like I had disappointed an important father figure. "Whoops, look at the time. Got to go. So sorry."

Presentation day came at the end of the week. Each of us had exactly five minutes. One by one my classmates went up to give their talks, only to be interrupted two-thirds of the way through and sat down because the five minutes were up. Then came my turn. I delivered three different proofs that the coffee cup was genus three, one of which was almost directly ripped from a proof technique my professor developed in one of his articles; I knew he would recognize it immediately. I finished my talk with finesse having ten seconds to spare, took a bow, and sat back down grinning from ear to ear. I was the only student in the entire class to finish my presentation in the five allotted minutes.

Grades came in the next week, and I received an A for the class. My peers were astounded. He had given me a failing grade on nearly every assignment and test for the entire semester. Sure, I did a stellar performance on my final presentation, but there is no way that the numbers would average like that. I told them the moment I realized he was so unprofessional as to be grading by impression as opposed to by rubric, that I knew he was also probably unprofessional enough to be susceptible to things like flattery. I just had to give him the IMPRESSION that I DID know what I was doing. And I won. THE END!

Robin Blankenship is an associate professor of mathematics at Morehead State University in Kentucky. Born and raised in the Appalachian Mountains, she obtained a BS in math at East Tennessee State University. There a professor changed her life forever by making her promise that she would take graph theory at her earliest opportunity. Next she obtained an MA in math at the University of North Carolina-Wilmington, which changed from a focus in number theory to a thesis in chaos theory after a professor caught her throwing her papers down the stairwell from the third floor late at night. The promise was fulfilled with a PhD in topological graph theory at Louisiana State University-Baton Rouge. Post-doctoral work in math education occurred at Appalachian State University, where she traveled as the Math Mobile delivering hands-on activities to grades two through five in addition to creating a variety of camps. Since then she has written a play called Last Fraction Hero *that toured to perform for over 32,000 students and she also loves to work with undergraduate research students. When not doing math, she sets out with her camping tent, cast iron for cooking, preparations for swimming, kayaking, and caving, and prefers no agenda so every day is improvisational. If she visits a city, then finding excellent restaurants is a must, because she loves food and she loves trying new things.*

26

A Walk in the Park Isn't Always a Walk in the Park

David Neel

Courtesy of Lina Shehu

Grey March afternoon sky fills most of the window, and the New Hampshire trees are still bare out there behind Ken Bogart, seated at his cluttered desk, looking at me. I'm lucky because I've got a job lined up. I'm unlucky because I've got news for Ken. You don't want news for your graduate advisor in March of your last year. Not about your thesis.

I'd found a shorter proof for the main result. We like short proofs. But when they get too short… "Yes, it doesn't quite seem like the main result of a thesis anymore, does it?" Ken said. But, with a gentle quiet laugh, he suggested we focus on a problem from the "To Do After Graduate School" list. That calm, and the quick pivot to a plan, dulled any panic or despair.

In my remaining months, we worked steadily, but in mid-August it was no great surprise to drive out of town, headed west to that new job, my grey 1984 Accord stuffed with all I had, which did not yet include a PhD diploma. Some eight months later, Ken summoned me to his winter home, which nestles, still, back against the wooded hills of Santa Rosa. One-week visit, smack before the holidays. One final intense blitz, Ken said, and we could complete the proof.

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I hoped he was right.

But by three days in, though we knew the problem more intimately, the long sessions at the whiteboard in his basement office had failed to produce the breakthrough. I felt claustrophobic, anxiety-ridden at the closeness. Each day, all day, we were together. Were we stuck?

That third night, Ken suggested a change: since tomorrow was predicted to warm into the high 50s, we would pack a picnic in the morning and hike up into the state park which bordered his backyard. He knew a nice clearing for some lunch, only a half-hour's walk. Talk through the problem, aloud, feet in motion, see if we could sketch out the missing pieces.

A restless night. Despite his kindness, and also because of it, I felt intense pressure. Over five years of my life, and it comes down to this. Morning finally broke. Standing in Ken's kitchen, at the island, each assembling our own sandwich which would then be placed into the pack. Maybe I would never finish. What if we just couldn't find the proof? I could hardly remember the right way to put together a sandwich, suddenly. When do I put the mustard in? Do I even like sandwiches? Is Ken watching me fail to make this sandwich?

And I wore the wrong shoes. Ken warned me there could be some mud, so I chose the cheap pair, brown fake leather, instead of my new runners. Why not? It's only going to be a half-hour hike. Then back again, but that'll be downhill, so it hardly counts.

The hike to lunch took closer to 45 minutes, but the weather sparkled and my flat feet didn't feel too bad. Most importantly, the shifting angles of the light, the mixed and faded shades of evergreen trees and grasses, the brown of mud and trail, the fact that we were watching our feet, and the sky, and the undergrowth, this all somehow focused our concentration into a pair of tiny sparks, dancing amongst dry tinder.

I almost worried when we stopped in the clearing. We had no new insight yet, but something loomed, felt nearer somehow. We had stated the problem with more clarity, or with a more promising perspective.

The sandwich, despite my earlier confusion, tasted good. We continued mentally circling the problem, but gently approaching, not forcing.

"You know," Ken said, repacking his pack, "we could just go back, but it seems like we're on the right track. Do you want to hike in a bit deeper and keep talking?" I ignored my aching fallen arches and nodded. Follow Ken's lead. He's the one who's been through this all before.

So walk on we did, which is when things began to go even better and also much, much worse. Though one would hardly call the day hot, the tempera-

ture had risen to the 70s, warmer than we had planned for. We began to consider, far too late, the water situation. One carries rather different amounts of water for a half-hour hike than one that will stretch into the afternoon. Also, I held my silence, but my feet were now delivering rather insistent pain messages to my brain, which resented these messages as a distraction from the urgent matroid issues at hand. Still, we were figuring things out.

It was around this time that we reached a split in the path. One branch would take us looping back down toward our origin. The other would lengthen the hike by perhaps another half-hour or so, according to Ken. Keep walking. Keep talking about combinatorics.

Still attuned to the nature surrounding us, Ken glanced awkwardly to catch a glimpse of a soaring hawk, and wrenched his neck. He complained little, but the rigid stillness of his head spoke clearly of pain.

We reached another split in the path. We may have made a joke about graph theory. Two paths, but the shorter one was marked: "Path Closed for Maintenance." Still two options, really: forward or back. We do not have a map. Ken is pretty sure he remembers where this forward path emerges. It should be fine.

Keep walking. Keep talking about combinatorics.

Another hour. Another split. The shorter path home, again, marked, "Path Closed for Maintenance." We are a ragged pair. Sprained neck. Wrecked feet inside cheap shoes. But at this point, can we really turn back? It feels as if this has become some elemental struggle. And we're still, somehow, talking about combinatorics. But now, we have it. It snapped into place. Now, we are sketching it out more fully, outlining it for each other, repeating, so that if only at least one of us can somehow walk or crawl free once more into that world outside this state park we can explain it to someone.

Emerge we do, at last, both of us, onto a winding neighborhood street far, far above his home. Dusk is falling, but we have the proof in our heads and, though our feet rage, at least from here it is all downhill.

The moral is clear: care for one another, keep walking, do not despair. One other moral: Ken's example, his kindness and generosity. He was a model mentor and a good man. I had hoped and expected more years and many more chances to thank him.

David Neel attempts to facilitate the learning of mathematical truths (but probably too often commits stultification by explication) at Seattle University. He earned his PhD under Ken Bogart at Dartmouth College. His curiosities include: graph theory, inter-

sections of mathematics and philosophy, knot theory, radical theology, inappropriate appropriations of mathematical objects, fiction, and math as a tool for conceptual development of (and action towards) a less dreadful world. He and his wife watch too much horror and are the proud parents of four cats: Zero, Sylvester, Marlowe, and Aki. David still has flat feet.

27

Just Don't Bomb the GRE

Amanda Ruiz

As I was finishing up my master's program in math and getting ready to apply to PhD programs, I found myself sitting on a plane next to a prominent mathematician, my dream thesis advisor at my dream school. He agreed to read my personal statement—he saw promise in me. He said, "You have a really strong chance of getting in. Just don't bomb the GRE." But that week, rather than study for the GRE, I had made the decision to attend a short course that he was teaching on my dream dissertation topic. He had even given me a research question to think about.

I was scheduled to take the math subject GRE a week after the short course ended. I hadn't studied at all. In fact, I had a lot of resistance to studying for the GRE. I had always done well on standardized tests. I'm pretty good at figuring things out on the spot, terrible at memorizing. But every time I opened that GRE study book, I felt like an impostor. It made me feel stupid. It didn't value my kind of smart. So, I devalued it the same way it devalued me. It had been five years since I took calculus, and for the past two years, I had been taking graduate-level math. My idea of a math problem was the type that came in a set of three-to-four problems that would take one-to-two weeks to figure out and write up. The idea of spending three hours completing 60 problems—most of which required a sneaky trick I was supposed to recognize and use instantly—seemed completely disconnected to the type of math I cared about. In particular, it had NOTHING to do with

graduate-level mathematics. My conclusion was that this exam is garbage and not worth my time.

That next weekend, my GRE didn't go well. Those words "just don't bomb the GRE" played on repeat in my head as I fumbled through three hours of shame. Still, I held up hope that I may have scraped by with a respectable score. I didn't need it to be great, just respectable. But my score came back that winter, and I had definitely bombed it. That meant that all my top choices for schools were out of the question. My backup schools were out, too. I ended up getting into two schools, and only one was offering funding. It was a school I almost didn't apply to. It was in a part of the country I didn't want to have anything to do with. I wasn't excited to go there, but it turns out that it was exactly where I needed to be.

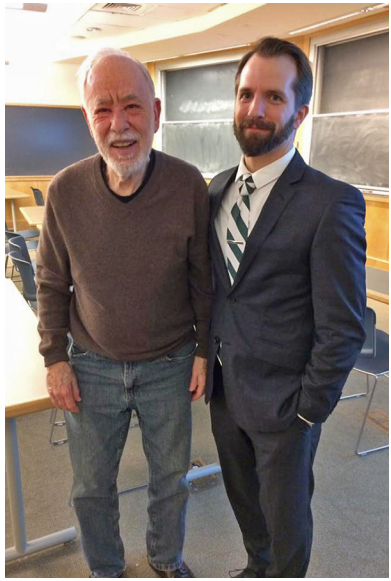
I didn't have the kind of mathematical community I had hoped for in grad school and left behind at my master's institution. But I had some remarkably supportive faculty mentors and administration within the department. They allowed me the flexibility to have and raise my child and also manage an increasingly complicated life. While I consider earning a PhD to be a huge accomplishment, I am most proud of the child I brought into this world, (and miraculously helped keep alive) while working towards my PhD. If I had been at my "dream school," it is unlikely that I would have been able to continue to work towards my PhD or, at the very least, finish in a timely manner while caring for my daughter. If I had gone to my dream school, my career trajectory would look different, and I would not have allowed myself to start a family until after tenure, which would have been biologically too late for me. So maybe, in a weird way, bombing that GRE was exactly what needed to happen.

Amanda Ruiz received a bachelor's degree from the University of California at Berkeley, where she created her own major focused on social movements. She also earned a master's degree in mathematics from San Francisco State University, and a PhD in mathematics from Binghamton University. Amanda believes that access to mathematics is a social justice issue that she has a responsibility to address as an educator. She does this through her commitment to making mathematics more accessible for women and students of color, and she considers service in this area to be an integral part of her job as an educator and an academic. Her prior research is predominantly in combinatorics and matroid theory, but her more recent research has expanded to include pedagogical studies. She is particularly interested in using math to study issues of social justice and investigating pedagogies that make mathematical spaces more inclusive—where those traditionally underrepresented in mathematics can thrive.

28

Sometimes When Your Hopes Have All Been Shattered

Nick Scoville



My first story begins during my senior year of college. As an ambitious student, eager to attend graduate school to earn my PhD in mathematics, I needed a good score on the GRE math subject test. Young and naive, I went into the test unprepared, having neither studied content nor the structure of the test itself. As a result, I scored in the 28th percentile, a score that was not good enough to get into any of the ambitious graduate programs to which I had applied. However, I was fortunate enough to be able to continue at my undergraduate institution towards a master's degree. I would then have two years to prepare for a PhD program.

During my first year as a graduate student, my friend Jim was also applying to graduate school, and he was planning on taking the GRE math subject test. Why not, I thought, try taking it again? But old habits die hard. Once again, without any preparation or study, I took the math GRE. Yet this time the results were very different. To my pleasant surprise, I scored in the 44th

percentile. This was an incredible improvement, especially as I had not studied. With a full year of study time left and a renewed sense of confidence, I outlined a detailed plan to study for the math GRE. I obtained every single practice test that exists and all the books that I knew of devoted to the math GRE. That summer, I spent three or four hours a day studying calculus, algebra, complex analysis, and everything else. By the time the day of the test rolled around, I was a machine, ready for anything the test could throw at me. I knew how the test was structured, its ins and its outs. I knew my strategy, and I knew what kinds of questions tended to be asked. In the words of Stan Bush, who provided the musical theme song to my GRE preparations, I had the touch. I had the power! Then I took the test—43rd percentile. I had devoted everything I had to slaying this beast, and I had done worse. I ultimately was accepted into Dartmouth's PhD program on the strength of my master's-level work and the recommendations of my professors—but my test-taking abilities had not helped my cause.

A year later, my first year in a PhD program, I saw Jim at a conference. While discussing our respective programs, the subject of the GRE came up. I had never told him or anyone else how I did on the test. I was too embarrassed. Now safely in grad school, I revealed my score to Jim. Prepared to hear about how his opinion of me had plummeted, Jim looked at me thoughtfully and spoke words that I will never forget: "It's too bad you didn't do better than you had hoped… but after having done worse, you still pursued graduate school. I actually have more respect for you than if you had you scored higher." I never thought about it that way, but after hearing Jim's words, I gained a great sense of accomplishment for having persevered through such a crushing setback.

Fast forward eight years. Now that I was a professor at a small liberal arts school and actively working on research with undergraduates, a major goal of mine was to obtain an NSF-funded grant to support a research experience for undergraduates (REU) program. Although I had little knowledge of REUs or what makes for a good proposal, I applied, but my proposal was rejected with mediocre reviews. For the next two years I continued to apply for REUs. My third attempt, while rejected, was rated "excellent," the highest possible score. These were the best reviews to date, so I was motivated to get serious. I received advice from my friends who have run REUs, and I studied their proposals. Then, I served on two NSF panels and began to understand the review process from the inside. I finally knew about REUs and the NSF review method. I had all the tools I needed to write a stellar proposal. This was definitely the year that we would get the REU grant. Again, in the words

of Stan Bush, I could win, if I dared. Then I received the reviews. My proposal was rejected, and I received my first "P" for poor, the lowest possible rating. The proposal was demolished, the worst reviews by far.

Once again, my very best efforts had resulted in total failure. I'm not able to tell you how this little story ends, as I am still trying to get my first REU grant. But my past experience tells me to keep trying. In the world of math, as in most of life, persistence pays off.

Nick Scoville received his bachelor's degree in mathematics from Western Michigan University and his PhD in algebraic topology from Dartmouth College. His research interests are in discrete Morse theory, digital topology, and persistent homology. He is currently the Joseph Beardwood III Chair of Mathematics at Ursinus College. When not counting holes, he loves to cook Indian food and BBQ while listening to old school country music, sew and tailor clothes while listening to Italian opera, and read classic literature. He is the father of five daughters and considers himself an amateur scholastic.

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The Compassion Is Life-Changing

Hortensia Soto

Photography by Desiree.

It can be difficult to imagine pursuing mathematics as a career when you are an immigrant, born in a two-room adobe home, and when your parents have a third-grade education. Yet, here I am. Research demonstrates that fifth grade is a pivotal year for young girls. It is at this young age that they develop attitudes, such as perseverance and confidence, needed to pursue STEM related careers. In this story, I paint snapshots of how I was able to develop such attitudes, which made mathematics attainable.

My dad, mom, older sister, and I immigrated to the United States when I was one-year old, and I grew up speaking Spanish at home with my parents and eight siblings. I went to Head Start preschool and kindergarten, but it was in the first grade that my teacher, Mrs. Steele, separated the students into an A and B group. I was in the B group because I did not know as much as my classmates did. I still did not know how to spell my name, so I went to the door where Mrs. Steele had our names posted to see how to spell my name. While Mrs. Smith, the other first grade teacher, decorated her door every month, Mrs. Steele kept our names up until I learned to spell my name—which did not happen until December. I will never forget the hug that she gave me once I did not need to go to the door.

Learning to spell and to define words was the biggest lesson of second grade. I remember spelling lace correctly and Mrs. Calvert saying, "Good, now use it in a sentence." I knew she knew that I was at a loss when she added, "It's on your dress." My mom made all my clothes and this dress had lots of things including buttons, polka dots, and a zipper. None of these resulted in a positive response as I pointed to them and looked for affirmation from Mrs. Calvert. She provided me with the hint of "It's at the bottom of your dress," but I translated "bottom" to mean "under," so I slightly lifted my dress to show my slip. The look on Mrs. Calvert's face was all I needed to know that my slip was not lace. Yet, Mrs. Calvert, with excitement, commented, "You are touching it." Finally, I learned what lace looked like, and I also learned what patience looked like. Giving up was not an option with Mrs. Calvert's patience.

In fifth grade, I had another teacher named Mrs. Calvert, whom I adored. One day she said, "Hortensia from now on, do not go to recess after lunch, instead come in to do extra work." I was devastated because I thought I had done something wrong, and I was scared that she would call my folks. For what seemed like an eternity, I went in after lunch and did extra work in mathematics, science, social studies, and phonics. Then one day she said, "Okay, tomorrow you can go back to recess, and starting tomorrow, you will be in the A group." Crying, I exclaimed, "I don't want to be in the high group." In denial, the following day I still went to Mrs. Calvert's room. Without saying a word, she gently took my hand, walked me to the high-group class, hugged me, and whispered, "You are going to be okay; you are ready." I walked into the A math group and … I *was* okay.

It was in high school that I discovered my passion for mathematics. Mr. Ingraham, who was my mathematics teacher all through high school, was ahead of his time regarding pedagogical approaches. He lectured for a few minutes, and we spent the rest of class time working together, constructing knowledge, and presenting our work. Mathematics did not come to me quickly, but I was willing to spend three hours a day working on and struggling with the homework. I very much appreciated this struggle because it was so much easier than working in the fields, which I did in the summers. For me, this struggle was play. By the time I was a senior, I was the substitute teacher when Mr. Ingraham was gone.

At the beginning of college, I had dreams of becoming a lawyer until my advisor commented that I did not need the second semester of calculus. To this I replied, "But we did not finish the book." Chuckling, he asked, "Don't you think you should be a math major?" I knew the answer was yes.

Becoming a mathematician has different obstacles that occur at different times for different people; overcoming these obstacles requires perseverance. I am lucky that I learned about perseverance at a young age from my parents and my teachers. Mrs. Calvert's belief in me and her compassion, opened doors and my mind to a life that I did not know existed. Fifth grade was pivotal for me because I started to have confidence in myself, recognized the value of hard work, and I began to envision a future that was not available to me in Mexico. At that time, I did not know that mathematics would be part of my future, but I started developing the skills and mindset to be able to succeed in mathematics.

We are not alone in the struggle to becoming a mathematician. We can always find someone who is patient, believes in us, and shows compassion. In working with prospective and practicing K–16 teachers, I remind them that teaching is about relationships, which requires compassion at every grade level. This can be life-changing.

Hortensia (Tensia) Soto is a professor at the University of Northern Colorado, School of Mathematical Sciences. Her research centers on the teaching and learning of undergraduate mathematics, where she adopts a lens of embodied cognition. She is a working member of the MAA and currently serves as the Associate Secretary. Necessities in her life include yoga, meditation, paying it forward, and spending time with her son, Miguel.

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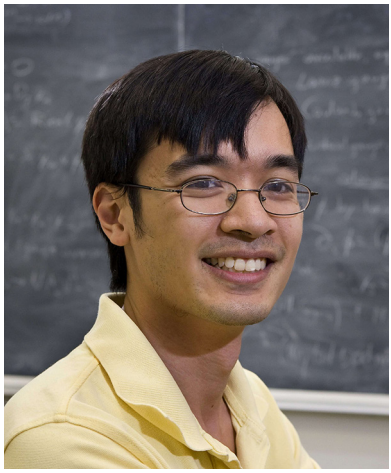
A Close Call
How a Near Failure Propelled Me to Succeed

Terence Tao



Courtesy of Reed Hutchinson/UCLA

For as long as I can remember, I was always fascinated by numbers and the formal symbolic operations of mathematics, even before I knew the uses of mathematics in the real world. One of my earliest childhood memories was demanding that my grandmother, who was washing the windows, put detergent on the windows in the shape of numbers. When I was particularly rowdy as a child, my parents would sometimes give me a math workbook to work on instead, which I was more than happy to do. To me, mathematics was an activity to do for fun, and I would play with it endlessly.

Perhaps because of this, I found my mathematics classes at school to be easy—perhaps too easy—even after skipping a number of grades. If a lecture was on a topic I found interesting, I would use the class time to experiment with the material, perhaps finding alternate derivations of some step the teacher did on the board, or to plug in some numbers to try out special cases and look for patterns. If instead I found the topic to be dull, I would doodle like any other bored student. In either case, I did not take particularly detailed notes, nor did I ever develop any systematic study habits. I would be

able to improvise my way through my homework and exams, for instance, by cramming through the textbook a few days before a final exam and perhaps playing a bit more with the parts of the class material that I really liked. It tended to work fairly well all the way up to my undergraduate classes. The courses that I enjoyed, I aced; classes that I found boring, I only barely passed, or (in two cases) failed altogether. (One class was a FORTRAN programming class in which I had refused to learn FORTRAN on the grounds that I already knew how to program in BASIC; the other was a quantum mechanics class in which we were warned well ahead of time that the final exam would require us to write a short essay on the history of the subject, which I totally ignored until the day of the exam, during which I still recall having to be escorted from the examination room in tears.) Despite this, I ended up graduating from my university with honors at the top of my class—but it was a small university with a tiny honors program, and in fact, there were only two other honors students in mathematics in my year!

When I entered graduate study at Princeton, I brought my study habits (or lack thereof) with me. At the time in Princeton, the graduate classes did not have any homework or tests; the only major examination one had to pass (apart from some fairly easy language requirements) were the dreaded "generals"—the oral qualifying exams, often lasting over two hours, that one would take in front of three faculty members, usually in one's second year. The questions would be drawn from five topics: real analysis, complex analysis, algebra, and two topics of the student's choice. For most of the other graduate students in my year, preparing for the generals was a top priority; they would read textbooks from cover to cover, organise study groups, and give each other mock exams. It had become a tradition for every graduate student taking the generals to write up the questions they received and the answers they gave for future students to practice. There were even skits performed (with much gallows humor) on hypothetical general exams with a "death committee" of three faculty that were particularly notorious for being harsh on the examinee.

I managed to brush off almost all of this. I went to the classes that I enjoyed, dropped out of the ones I did not, and did some desultory reading of textbooks but spent an embarrassingly large fraction of my early graduate years messing around online (having discovered the World Wide Web in my first year) or playing computer games until late at night at the graduate dormitory computer room. For my general topics, I chose harmonic analysis---which I had studied for my master's degree back in Australia---and analytic number theory. Feeling that analysis was my strong suit, I only spent a few

days reviewing real, complex, and harmonic analysis; the bulk of my study, such as it was, was devoted instead to algebra and analytic number theory. All in all, I probably only did about two weeks' worth of preparation for the generals, while my fellow classmates had devoted months. Nevertheless, I felt quite confident going into the exam.

The exam started off reasonably well, as they asked me to present the harmonic analysis that I had prepared, which was mostly material based on my master's thesis and specifically on a theorem in harmonic analysis known as the T(b) theorem. However, as they moved away from that topic, the shallowness of my preparation in the subject showed quite badly. I would be able to vaguely recall a basic result in the field, but not state it accurately, give a correct proof, or describe what it was used for or connected to. I have a distinct memory of the examiners asking easier and easier questions, to get me to a point where I would actually be able to give a satisfactory answer; they spent several minutes, for instance, painfully walking me through a derivation of the fundamental solution for the Laplacian. I had enjoyed playing with harmonic analysis for its own sake and had never paid much attention as to how it was used in other fields such as PDEs or complex analysis. Presented, for instance, with the Fourier multiplier for the propagator of the wave equation, I did not recognise it at all, and was unable to say anything interesting about it.

At this point, I was saved by a stroke of pure luck as the questioning then turned to my other topic of analytic number theory. Only one of the examiners had an extensive background in number theory, but he had mistakenly thought I had selected algebraic number theory as my topic, and so all the questions he had prepared were not appropriate. As such, I only got very standard questions in analytic number theory (e.g., prove the prime number theorem, Dirichlet's theorem, etc.), and these were topics that I actually did prepare for, so I was able to answer these questions quite easily. The rest of the exam then went fairly quickly as none of the examiners had prepared any truly challenging algebra questions.

After many nerve-wracking minutes of closed-door deliberation, the examiners did decide to (barely) pass me; however, my advisor gently explained his disappointment at my performance, and how I needed to do better in the future. I was still largely in a state of shock—this was the first time I had performed poorly on an exam that I was genuinely interested in performing well in. But it served as an important wake-up call and a turning point in my career. I began to take my classes and studying more seriously. I listened more to my fellow students and other faculty, and I cut back on my

gaming. I worked particularly hard on all of the problems that my advisor gave me, in the hopes of finally impressing him. I certainly didn't always succeed at this—for instance, the first problem my advisor gave me, I was only able to solve five years after my PhD—but I poured substantial effort into the last two years of my graduate study, wrote up a decent thesis and a number of publications, and began the rest of my career as a professional mathematician. In retrospect, nearly failing the generals was probably the best thing that could have happened to me at the time.

My write-up of my general exams experience is still available online. I have been told that it has been a significant source of comfort to the more recent graduate students at Princeton.

Terence Tao was born in Adelaide, Australia in 1975. He has been a professor of mathematics at UCLA since 1999. He has received a number of awards, including the Fields Medal in 2006. Terence Tao also currently holds the James and Carol Collins chair in mathematics at UCLA.

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Oh My Darlin' Clementine

David Taylor

Courtesy of David Taylor

To be honest, I didn't really struggle much in high school or college; the biggest "struggle" during that time was figuring out what I wanted to do next, and, thankfully, that was helped along by some of my teachers and mentors. I taught myself computer programming in high school (first BASIC and Visual Basic, and then more advanced languages such as Java, Perl, and PHP), and I imagined myself being a programmer for the rest of my life. It was my senior year of high school when my calculus teacher recognized a budding interest inside of me of which I was not yet aware—he gave me a video (on VHS) about fractals, specifically, the Mandelbrot and Julia sets, which I found fascinating! My planned computer science major in college turned into a double major.

Then, while working on my undergraduate degree, my mathematics major advisor noticed my interest in helping others with mathematics and suggested that I consider going to graduate school. Being a first-generation college student, I had no idea what this "graduate school" was, but I remember being told that most mathematics PhD programs are "free" in that mathematics graduate programs tend to waive tuition for their students and even offer a small stipend to teach courses or do research. The prospect of being

paid to learn more was very exciting to me! Rather than just finishing my degree, ending my stay in higher education, and starting a career as a software engineer, I thought that staying in school, getting an advanced degree, and deferring the task of joining the real world job market sounded very attractive.

Graduate school in mathematics can feel *very* different from an undergraduate program, especially since there can be large differences between programs at the undergraduate level. Having come from a small, private, liberal arts program, I had a strong aptitude for problem solving, critical thinking, and knowing how to learn. What I lacked was depth when it came to subdisciplines within mathematics. The first semester in graduate school was a challenge, but not insurmountable; the entirety of my undergraduate abstract algebra course was covered in the first three weeks of graduate-level algebra, the complex analysis content that I had seen while auditing a course during my undergraduate years barely appeared in the graduate-level complex analysis course, and I had not seen any point-set topology yet at all! Thankfully, all three of my graduate professors that first semester cared about their content, their courses, and their students. The regular routine of homework and tests in those early courses was familiar to me. Succeeding that first semester was only a matter of putting in the time—I already knew how to learn and work through this.

In the following spring semester, however, the stars suddenly were *not* aligned, and one of the most difficult moments arose. The course was differential topology. The setting was a large room, occupied only by myself and the professor. A chalkboard was affixed to the wall at the front of the room, its slate devoid of writing. On the table nearby sat a hat with six slips of paper and … a box of clementines.

Rather than collecting homework and providing written feedback for that class, the professor had assigned exactly six homework problems and asked each student to come into the graduate study lounge, one at a time, to present a solution to a randomly-selected problem (the six slips of paper in the hat corresponded to the six problems). Class with this professor left much to be desired; each class session began with a review of the previous day, and that review period lasted nearly three-quarters of the class. It felt as though we weren't learning anything at all. Moreover, I cannot for the life of me recall having a textbook in that class. How were we supposed to do those six problems? The first time I had to present a randomly-selected problem, I got lucky and drew number one or two; I do not recall which, but I do know they were two "easier" problems that I knew how to do. Still, not being one

to enjoy public speaking, I was nervous but managed to do well. The professor then offered a clementine as a parting gift to eat and enjoy.

The second problem set, again with six problems, was even tougher. As a class, we managed to convince the professor to change his problem selection process slightly. This time, we were allowed to randomly pick two problems from the hat, and then choose which we wanted to do. In preparation for the dreadful day approaching, I really learned that, when allowed, collaborating and discussing problems with friends is an amazing way to learn. I hadn't needed to do this for most of my education, but now I had reached the point where this was essential. By myself, I only knew how to do two problems and only *really* understood one of those. As a group, we were able to talk through ideas and solutions to the point that I knew how to do—and thoroughly understood—problems one, two, three, and five. By the time the oral review day arrived, I even had a vague understanding of problem six.

To borrow a similar phrase from Washington Irving, it was a dark and stormy day. The vividness of the clementines sitting in the box on that table stood out to me. I put my hand into the hat and grabbed two slips of paper—I got to choose between problems four and six; at this point, the choice was easy—I would do as well on problem six as I could, and then leave quickly. I managed to explain enough, I suppose, and after I was done, the professor looked at me and asked `"how does four out of five sound for a grade?" Very quickly, I said, "Great!" and moved to leave. "Clementine?" the professor asked, to which I politely said, "No, thank you," and left the room. I was content. I had been unlucky in randomly choosing my problems, but I managed to scrape by with a decent grade on the homework.

I learned a lot that semester about the importance of collaborating with my friends and the value of trying the best that you possibly can in each moment. Acting on these two lessons while leaning on professors and advisors to help me through challenges helped shape my path. I share these same lessons with my students every semester.

Oh, and I still do not like clementines.

David Taylor is an associate professor of mathematics and statistics and chair of the Department of Mathematics, Computer Science, and Physics at Roanoke College in southwest Virginia. He earned his undergraduate degree in mathematics and computer science in 2002 from Lebanon Valley College, his master's degree in mathematics in 2004 from the University of Virginia, and his PhD in mathematics in 2007, also from the University of Virginia. He started at Roanoke College in the fall of 2007 and quickly

fell in love with blending teaching, scholarship, and service. Today, David splits his time almost evenly between teaching and administrative duties, with those administrative tasks focused on improving education for undergraduate students. He has served as Program Chair and Section Chair for the Maryland, DC, and Virginia Section of the MAA and enjoys collaborating with colleagues on research projects designed to help undergraduate mathematics students. In his spare time, David enjoys spending time with his dog Lilly, cooking delicious food (including fresh pasta), reading science fiction, playing intense and long board games, and writing long sentences.

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A Dream Almost Deferred

Emille Davie Lawrence

Maybe you should consider doing something else with your life.

This was the advice that was given to me by my academic advisor after I completed my first year of graduate school. It was the summer, and I had just found out that I'd failed both qualifying exams—analysis (which included real and complex analysis) and algebra—that I'd taken weeks prior. She asked no questions about my preparation. No deeper dig into what might have happened. No "if at first you don't succeed" bit. Just a direct suggestion that I should consider abandoning my career goals because of a failed first attempt. What I heard was, "Nice try, kid. Now, run along."

I could have done just that. I could have run away from my dreams, my head hanging in shame because this person who, of course, must know everything there is to know about navigating a PhD in mathematics said that I didn't measure up. I could have questioned whether I belonged at a large state university having finished my bachelor's degree at a small liberal arts college. I could have let her comments dim the lights on the future that I saw for myself.

But, fortunately, I did not do any of those things.

What I did do was thank her for her time, and then I began to make steps toward finding a new advisor. My new advisor was a man (I mention this because I suspect that I was paired with the first advisor because we are both women), and more importantly, he believed in me despite my not-so-great first showing. This self-advocacy was a pivotal moment for me not only in graduate school, but in my life as a whole.

I began to study again, this time with one-on-one help from some very generous professors. They each sat down with me once per week to help me flesh out the gaps in my understanding of both analysis and algebra. Then, we'd go over concepts, and I'd practice solving old qualifying exam problems and present my solutions. It was a slow process, but each week my comprehension got a little better and the problems became a little more intuitive. Now, as a professor, I understand what a tremendous offering of their time they made—and it was all because they believed that I could succeed.

And succeed, I did. I eventually passed all three of my qualifying exams (including topology, which I easily passed on my first attempt), and the rest is, as they say, history. My struggles with qualifying exams shaped my approach to student interactions. I will never write a student off or underestimate their ability to learn. I will never separate my students into the categories of those who can do math and those who cannot—only those who aspire to do the work that it takes to succeed in math and those who do not. I try to give generously of myself to my students and show them the grace that was shown to me so many years ago. In the end, this is all that any of us can do to make room in the mathematics community for everyone.

Emille Davie Lawrence is a term assistant professor of mathematics and statistics at the University of San Francisco. She earned her BS in mathematics from Spelman College and her PhD in mathematics from the University of Georgia. She has also been a postdoctoral fellow at the University of California, Santa Barbara and an assistant professor at California State Polytechnic University, Pomona. Her research focuses on topological properties of spatial graphs, and she is involved in several national programs aimed at broadening participation in the mathematical sciences. Emille enjoys speaking about mathematics to people of all ages and believes strongly that mathematics should be accessible to everyone. Her non-professional life is filled with music and other performing arts and spending meaningful time with her husband and two children.

IV

What Do I Do Now? What Happens Next?

As advisors of students in college, the editors frequently come across students who ask "What can you do with a degree in mathematics?" This is really hard to answer, because, in a way, "anything" is not too far from the truth. At its very core, studying mathematics helps students become better at critical thinking and problem solving, two skills that are essential in today's world. Since the editors of this book are all professors ourselves, it can be daunting to talk to students about the so-called real world and what jobs are really out there.

Students struggle with finding themselves as a mathematicians and what to do after they graduate; here are some stories from people who felt that struggle and resolved that conflict in various ways.

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Mathematics, Beauty, and Creativity

How I Learned to Stop Worrying and Love Mathematics

Victor Piercey

Courtesy of Kent Sun

In high school, as I was finding my academic way, I felt that math was a very conformist subject. I was interested in history and philosophy, areas where there was room for creativity and interpretation. Math presented itself as a subject in which success meant following orders and doing what you were told. Where was there room for interpretation and creativity in memorizing and reciting the derivation of the quadratic formula? At least the recitation of a soliloquy from Macbeth had room for interpreting and expressing the lines. In my teenage angst, I rebelled and rejected what I felt were faulty premises in mathematics as an intellectual endeavor. I felt that mathematics was a barrier to my sense of self as creative and humanistic.

All of this changed when I took calculus in college.

To be sure, I was not required to take calculus. My first major was history, which I still sometimes think of as "the one that got away." When I arrived on campus at orientation, ready to reinvent myself as a serious student, one of my first tasks was to take a math placement exam. I don't remember

recognizing much on this exam, but I do recall reasoning what function notation must have meant based simply on context. Somehow, despite failing my last semester of precalculus in high school, I placed into calculus. What this meant, as a history major, was that I placed out of the university math requirement.

Shortly after my second year in college, my interests and career aspirations shifted. I had been studying Russian and Soviet history. What seemed more exciting was the Russian present. It was 1995, and the former communist countries of Eastern Europe and Russia were struggling through a transition to capitalism that faced acute difficulty in Russia. This made me potentially interested in pursuing a PhD in economics. I changed from a major in history to an interdisciplinary major in humanities, which provided me with sufficient flexibility to prepare for such a graduate program.

My interest in economics drove an interest in mathematics. To this point, I had been sufficiently successful in college to develop the hubris that I could actually succeed in calculus. The flexibility of my new major allowed me to take calculus. I wanted to take the honors calculus sequence, since the course descriptions indicated that the honors courses focused more on theory. I thought (rightly, as it turned out) that a more theoretical foundation would make it easier for me to improvise and apply calculus to problems I may encounter in my future intended studies. I also thought it would help get into stronger graduate programs in economics.

As I predicted, my internal motivation helped me succeed in the honors calculus courses. There were moments of difficulty. I specifically remember trying to use the definition of the derivative to differentiate a cubic polynomial and having trouble with the bookkeeping. I felt that this was not what I had signed up for. All the old thoughts that math was about meaninglessly following orders resurfaced. But I looked to my own internal reservoir of motivation to pull through. Much of the time, my experience in calculus was driven by the connections I was making to economics. That made the mathematics real to me.

In the first calculus course, I recall the professor using the word "beautiful" to describe something discussed early in the course. That caught my attention as it puzzled me. I wondered what it meant for mathematics to be beautiful. A couple of months later, on a Greyhound bus, as I was reading ahead in our calculus textbook, I saw the fundamental theorem. For the first, and far from the last, time, I found beauty in mathematics.

The most significant turning point for me occurred near the end of the second semester of calculus. On an exam, I wrote a proof for convergence of a series that the professor marked as "creative." His solution relied on a te-

dious application of the ratio test. I made (and proved) a simple observation (that $n^n > n!$) which reduced the problem to a simple comparison test. The fact that there was room for "creative" solutions to mathematical problems stunned me. That I had produced such a thing stunned me more. My desire to be creative and expressive connected with mathematics, and that shift in my sense of identity lasted.

After finishing college, I ended up going to law school instead of a graduate program in economics. Based on my experience as an intern at a firm in Moscow, I felt that law was the quickest way to get directly involved in the transition. However, once I started working in a law firm, I learned quickly how miserable I was in this profession. Each day felt hopeless, and I searched for a way out. Some ways out were darker than others.

In this environment, I recalled the joy that followed intense struggle with mathematical problems. I remembered how it felt to find beauty in mathematics, and how it felt to have a solution considered creative. Since I had taken additional math courses, such as real analysis and linear algebra, as electives, I was able to negotiate my way into a master's program which would serve as an opportunity for me to transition from one career to the next. Now I am a PhD holder (dissertation in algebraic geometry) with a tenured position as an associate professor. My days as a lawyer seem a distant memory, and in some small way, I feel that mathematics saved my life and my soul.

Victor Piercey received an interdisciplinary BA in humanities from Michigan State University in 1997, a JD from Columbia Law School in 2000, a MS in mathematics from Michigan State University in 2006, and a PhD in mathematics from the University of Arizona in 2012. His dissertation was in algebraic geometry. Since completing his PhD, he has been teaching at Ferris State University in Big Rapids, Michigan where he was granted tenure in 2017 and is an associate professor in the mathematics department. His primary work at Ferris has involved the development of a two-semester sequence of general education courses entitled Quantitative Reasoning for Professionals. He also led the development of the actuarial science program, in which he developed and taught the Theory of Interest course. In 2018, he was appointed Director of General Education. His teaching interests are very broad, including the integration of ethics with mathematics, the use of inquiry-based learning, interdisciplinary collaboration in course design and in teaching, and the use of role-playing games similar to Reacting to the Past. In his spare time, he reads books on history, particularly Russian and European history.

34

Five Dollars

Colin Adams

When I was a graduate student at the University of Wisconsin-Madison, I was taking a couple of seminars in different areas, trying to figure out what I should work on. I knew I was interested in topology but as to what aspect, I had no idea. One of the seminars was on hyperbolic 3-manifolds. This was a hot new area of research, having recently been invented by Bill Thurston. Three-manifolds are spaces that are locally 3-dimensional, like the spatial universe we live in. We say a 3-manifold is hyperbolic if one can assign a way to measure distance in the space that is negatively curved, and if a manifold is hyperbolic then it has a unique hyperbolic volume associated with it. These ideas were revolutionizing low-dimensional topology. Jim Cannon, who was doing some work with Bill Thurston, was running the seminar on the Thurston notes, Xeroxed copies of notes mostly taken by Thurston's students for a course he offered in the late 1970s. It was an exciting time.

Knots can also be hyperbolic in the sense that you associate to the knot the 3-manifold that is obtained by removing the knot from space, the so-called complement of the knot. At that time, the hyperbolic volume was known for only one knot, the figure-eight knot. So, one day, Jim Cannon announced, "I will give five dollars to the first person who can find the hyper-

bolic volume of the 5_2 knot, which is the next hyperbolic knot in the table."

I was a starving graduate student, so I said to myself, "Okay! I am going to solve this problem!" And I began working on it. That was all well and good except I couldn't seem to make any progress. But I didn't give up. I continued working on it for a month, which, at the time, was the longest I had ever worked on a problem. (Nowadays, I know that's nothing, but back then, it seemed an immense amount of time.)

And then another member of the seminar, a postdoc named Dennis Stowe, announced that he had a solution. I was devastated. All my effort was for naught. At that point, I seriously doubted if I would ever become a mathematician.

But at the next seminar meeting, when Dennis showed us his solution, I realized what I had been doing wrong. And his method only worked on the one knot. My method, once repaired, worked on all of the other knots in the table. So, he got the $5.00, but I got a PhD.

Since then, my entire career has been built around knots, hyperbolic 3-manifolds and the connections between the two. All from just trying to win $5.00.

Colin Adams received his PhD from the University of Wisconsin-Madison in 1983. He is the Thomas T. Read Professor of Mathematics at Williams College and has published over 60 research articles, 100 expository articles and stories, and eight books, including The Knot Book, Introduction to Topology: Pure and Applied, Riot at the Calc Exam, *and* Zombies & Calculus.

35

Am I "Good Enough"?

Christine von Renesse

Studying at the university in Berlin, Germany, I was planning to become a K–12 school teacher. At first I was interested in teaching music in high school but then changed my major to teaching mathematics in elementary school—but that is a different story for another day. It is enough to know that I unexpectedly fell in love with mathematics and decided to also pursue my Diplom (master's) in mathematics in addition to my teaching degree. I had to work hard to make sense of mathematics, and I truly enjoyed the late-night homework sessions with my math friends, the hours spent at the library, and the satisfaction of figuring "it" out. Maybe I fell in love as much with the challenge as with the subject itself.

Having finished my Diplom in differential geometry, I wasn't ready to stop learning mathematics and go into teaching. Changing careers in Germany is unusual and the idea of committing to being a teacher for the next 40 years was truly terrifying. My friend and role model in mathematics (I called her my "math mama") suggested that I could go for my PhD. The thought was scary and exciting and I made an appointment to talk to the professor who knew me the best. I had taken five courses with him, asked him questions in numerous office hours, and he had supervised my Diplom thesis (final grade: A).

I was nervous during the meeting, as I was during most meetings with him. Professors in Germany convey a lot of authority and I was caught between simultaneously respecting him and wanting his approval, and trying to be an independent, rebellious person. During our meeting, he frankly told me that, even though I had only received good grades, he didn't think that I was "genius" enough to pursue a PhD in mathematics. Now, looking back, I wonder if he would have given similar advice to a male student? I don't remember the exact words said during the meeting, but I remember the feeling of defeat and doubt.

It would have been the easiest path to just finish the teaching degree and practicum, and to start teaching in an elementary school in Berlin. In fact, the education professor after my final oral exam practically begged me to go teach in the school system because many "good math teachers" find other jobs that pay better or are more valued in the public eye instead.

But I knew that I wanted something "more," that I wasn't yet done, learning and challenging myself at the university level. To stay at the university and make money while I finished my teaching degree, I joined a project at the university that worked on online mathematics education. The job included programming, designing mathematical tasks, working with databases, etc.

During my work with the online education project I stepped out of the role of a student for the first time. I collaborated with staff and professors, traveled to other universities, and formed clear opinions on my own. I also struggled with the leadership of the project and discovered a level of university politics and rivalry I had not been aware of before. As difficult as it was, the conflicts allowed me to work on communicating and justifying my pedagogical ideas, standing up for myself, and handling criticism. I used this time to build confidence in myself and learn more about the pedagogy of mathematics. Still, I missed the simple "right or wrong" nature of pure mathematics and, of course, I dearly missed teaching real students in a real classroom.

It took me two years to distance myself from the advice I had been given, and to find the confidence to pursue what I wanted. I visited my "math mama" who was then working in the U.S. Her "of course you can do this!" attitude had supported me during my studies in Germany and once again she motivated me to go for what I wanted. She helped me apply for a PhD program in Massachusetts, and just a few months later I was accepted and emigrated! Moving to a different country allowed me to redefine who I was and question many cultural beliefs and stories that I had bought into. Every

year during the PhD program I would ask myself: Am I still doing what I truly want to be doing? And every year the answer was a clear YES.

Moving to another country also helped me find a type of job that I don't believe exists in Germany. I am now a full professor at a small public university in Massachusetts and my responsibility is to teach four courses each semester. I teach future K–12 teachers and spend lots of time in K–12 classrooms, collaborating with teachers. I also (re-)discovered my love of music and dancing, and I teach mathematics for liberal arts courses that combine mathematics with the arts (see www.artofmathematics.org)! While I don't have to do research and publish for my current position, I truly enjoy doing mathematics, writing papers about teaching, going to conferences and facilitating workshops. All the aspects of my previous studies and interests have finally come together and make sense.

Christine von Renesse is a passionate teacher, who loves teaching at all levels—from elementary school through college. She uses open inquiry techniques in all her classes, believing that this is the most effective and enjoyable way of learning and teaching. Her students learn to take responsibility, to think independently and to enjoy the endeavor of challenging questions with growing confidence. The mathematics for liberal arts classes at Westfield State University allow Christine to bring her passion for dance and music into her mathematics life, exploring the deeper connections together with her students. In the mathematics courses for future K–12 teachers she brings her students into the classrooms, collaborating with the teachers in finding effective ways to explore mathematics together. In her free time Christine loves to explore nature, sing in harmony and go dancing, especially with her daughter. Christine has a master's degree in elementary education, a minor in music and a master's degree in mathematics from the Technical University Berlin, Germany. After receiving her PhD in mathematics at the University of Massachusetts, Amherst, she is now a professor at Westfield State University.

36

My Journey from Slippery Rock to Duluth

Joe Gallian

Courtesy of William Higgins

My journey from Slippery Rock State College in Pennsylvania in 1966 to the University of Minnesota Duluth in 1972 had quite a few twists and turns. In 1966, I received TA offers from Minnesota, Kansas, Purdue, and Michigan State. About a week before the deadline for accepting offers, I selected Minnesota. A few days later, Kansas called to ask me if I would be willing to come to KU on a five-year NASA Fellowship. I was delighted to accept.

Because the Rock was almost exclusively a school for the preparation of K–12 teachers, my course work there did not adequately prepare me for graduate-level courses. Instead, I took courses intended for juniors and seniors. Fortunately, I had a charismatic abstract algebra teacher named Lee Sonneborn for both semesters. I was so enthralled with that course that I took an independent study course in permutation groups and participated in a weekly seminar on infinite group theory. By the middle of my second semester, I decided to do a PhD thesis on infinite groups under Sonneborn. This plan abruptly changed a few months later when Sonneborn moved to Michigan State. Disappointed, but not deterred, I decided that I would do a thesis with Dick Phillips, who was another infinite group theorist participating in the infinite group theory seminar. But it was not to be. In the fall of 1967, Phillips told me that he would be going to Michigan State the next year.

The departure of both of my potential thesis advisors prompted me to apply to grad school at Michigan State, Utah, Illinois, and Notre Dame, all of which had infinite group theorists. When I received a three-year fellowship from Notre Dame, I accepted. Shortly after arriving at Notre Dame, I approached the infinite group theorist about working with him. As luck would have it, he told me that this was his final year at Notre Dame. The next best option was to work with Warren Wong, who was one of hundreds of people working on the classification of finite simple groups. Wong agreed to take me on, but he said he would be on sabbatical the next year in New Zealand and I was welcome to join him there. This did not appeal to me and my wife, so I declined. That left me with three options: transfer, abandon group theory, or do a thesis with Karl Kronstein, whose only publication was on representations of finite groups, a subject about which I knew nothing. I opted for the latter.

In the summer of 1969, Kronstein gave me a research paper on a new concept called the "breadth of a finite p-group" to study. The paper had some nice results and an appealing conjecture that Kronstein thought could be attacked using group representations. After six months of learning some representation theory and trying to apply it to the breadth problem, I had made no progress. I was so discouraged that I called the Slippery Rock department head about the possibility of leaving Notre Dame with just a master's degree and teaching there. He said he could not promise anything but encouraged me to stick it out at Notre Dame. I also talked to Kronstein about finishing the master's degree and getting a job. He told me that it was not unusual for people to make little progress in the first six months of thesis work. I ended up compromising by not giving up on a PhD, but giving up a representation theory approach to the breadth problem. In short order, I began to make substantial progress on the problem, and I even came up with a unrelated problem of my own that led to published results that generalized several well-known theorems on finite p-groups.

From then on, everything went smoothly until I applied for jobs in January 1971. Prior to 1971, Notre Dame PhDs had done well on the job market, but two things converged in 1971 to change employment opportunities dramatically. During the 1960s, the U.S. responded to the Cold War and the Russian success in putting satellites and people in space by pumping an enormous amount of funding into graduate study in STEM fields (indeed, my KU and ND fellowships were part of this). At the same time, men could avoid the military draft for the Vietnam war while in college. These had the effect of funneling large numbers of people into PhD programs. Although

everyone knew a bad job market was coming, it was not clear exactly when and how hard it would hit. The effect on me was that the 25 or so job applications I sent out in January did not generate a single response. So, it appeared I would be an unemployed new PhD in the fall of 1971. To my great relief, Notre Dame offered me a one-year postdoc.

In December of 1971, I sent job applications to 145 schools. By March, only four had expressed some interest, and none of these four offered me a job interview. In May, Notre Dame once again rescued me by offering another one-year postdoc. To this day, I shudder to think of how my career would have gone without those postdocs.

A few weeks later, I received a call from the University of Minnesota Duluth saying that a person who had just accepted a position there decided to stay where he was, and they asked if I was still interested in their job. I said that I was sure that Notre Dame would be happy to release me from the postdoc if I received an offer (which I confirmed the next day). About a week later, I interviewed at UMD and, a month later, accepted an offer. Forty-six years later, I am still teaching, writing, and engaging students in research.

In hindsight, I cannot imagine any of those other 144 schools being a better fit for me. I do not believe in destiny, but I do believe that good fortune often prevails when you keep up the struggle!

Joe Gallian received a PhD from Notre Dame in 1971. He has been at the University of Minnesota Duluth since 1972. He is the author of the book Contemporary Abstract Algebra (9th edition) *and coauthor of the book* For All Practical Purposes (10th edition). *His research interests include groups, graphs and combinatorics. He has published more than 100 articles and given over 500 invited lectures at colleges, universities and conferences. He has directed summer research programs for undergraduate students since 1977. He has received the Mathematical Association of America's Gung and Hu Award for Distinguished Service, the MAA's Haimo Award for Distinguished Teaching, the Carnegie Foundation Minnesota Professor of the Year Award, and awards for teaching and research from the University of Minnesota. He is a past President of the MAA, a Fellow of the American Mathematical Society, and was a long-time co-director of Project NExT. Gallian taught a humanities course on the Beatles for thirty-three years at UM Duluth.*

37

Failure By the Numbers

Dominic Klyve

My reality seemed dominated by numbers, organized in my mind in a neat table:

Photograph by Liam Klyve

6	The number of years I thought I had been working assiduously to craft the perfect CV and set of skills to ensure I could be employed as a mathematics professor.
10	The level of certainty felt by my spouse and I, on a one-to-ten scale, that we did not want to raise children in the area of the country in which we were living, and thus, that I needed a new job.
25	The number of universities with positions for which I was well-qualified, and the number I applied to, with carefully customized applications and cover letters for each.
60	An estimate to be sure, but a reasonable one, of the number of people I had talked to during in-person and on-campus interviews. Put another way, the number of people I had looked in the eye while I asked them to hire me.
0	The number of schools willing to hire me that year.

I was devastated. And embarrassed.

In 2009, I knew that for family reasons I needed to find a new job. While I loved the small liberal arts school where I was teaching, my family was unhappy in Wisconsin, and my partner and I both wanted to raise our children somewhere else. Because I was the only one of us working full time, I felt responsible for finding a new job, allowing us to move. I therefore updated my application files and started to send cover letters to schools across the country.

Things seemed to go well at first. I interviewed with several schools at the Joint Mathematics Meetings, and had telephone interviews with others. The process continued as I hoped it would when I was flown across the country for on-campus interviews. But the course of true employment never did run smooth.

There is an annoying time-asymmetry in the academic job search process. If you are the chosen candidate, you are the first to find out the results of the search. Other candidates are left to wait. The process makes sense—a school shouldn't reject someone until they're sure they can't get their first choice—but it does leave the unchosen candidates in an information-less state of waiting.

That winter, I waited a lot, knowing with each passing day that the likelihood I'd be offered a job was less than it had been the day before. By the time I received the last rejection, it was less a shock than a final confirmation of my failure.

I had failed my family, and I couldn't escape the feeling that my failure had condemned us to at least another year in an unpleasant environment. And, given that I now had hard data that no one wanted to hire me, it was hard to see a reason why the next year might be different.

I had consoled myself after past failures by telling myself that I hadn't tried as hard as I could, that I could have succeeded if only I had worked a bit more. But I knew this was a lie. The truth was that I had tried hard, and in fact I had worked hard for years trying to build a set of accomplishments that I thought would lead a school to want to give me a job.

I grieved (all five stages), but of course I eventually realized that my only option was to try again. One of the (very few) nice things about failure is that it often lays the groundwork for future success, and this turned out to be the case for me. Because I lacked confidence that I could find another academic job, I decided I could increase my chances of moving my family if I broadened my search, and so I started studying for and taking actuarial exams.

I decided I would try just one more time to land an academic job, and if I failed, I would become an actuary. The next year, I once again started to

send out applications (with, I remember well, considerably less excitement and optimism than I'd had the year before). I'm very happy to report that the second year went far better. I had more than one job offer, and I ended up accepting a position at Central Washington University, where I found myself in a wonderful department. I found out later that one of the reasons I ended up high on CWU's list was my experience with actuarial exams. Oddly enough, my ultimate success had sprung almost directly from my failure.

Dominic Klyve is a professor of mathematics at Central Washington University. He is the author of more than 40 papers in number theory, the history of mathematics and science, and applied statistics. His interdisciplinary works have appeared in journals ranging from Gastrointestinal Endoscopy *to* Shakespeare Quarterly. *Dominic has been nationally recognized for promoting the use of primary sources in the teaching of mathematics, and currently serves as a co-principal investigator on a $1.5 million grant from the National Science Foundation to develop classroom materials for this purpose. He was a 2014 winner of the Mathematical Association of America's Alder Award.*

38

How I Learned to Research Like the Incredible Hulk (or I'm Always Angry)

Robert Vallin

Courtesy of Robert W. Vallin

In the early 1990s, jobs were difficult to find, and my job search was torturous. I answered more than 100 ads, went to the 1992 Joint Mathematics Meetings (that year in Baltimore, MD) as a freshly-minted PhD, and participated in the cattle call known as the Employment Center. And then I waited, and waited, and waited. My first on-campus interview was in April. But the fates were kinder to me than others. In August, I was bright-eyed, bushy-tailed, and starting a tenure-track job. I landed at what seemed like a good, if not great, school for me—a mid-level state school with a four-four teaching load and "professional development" rather than research expectations.

I was okay with the teaching load, despite its heaviness. Service would work out—I had been involved in lots of service as an undergraduate and graduate student. But there was the third part. I did not have to do research *per se*, but I *wanted* to do it. Since the first time I figured something out that had never been figured out before and then saw my name in print, I wanted to do it again. I wanted to be like my thesis advisor and the bigwigs he

knew. But how? Yes, I had written a thesis and published it as two papers. However, my results had come from meeting with my thesis advisor, having him suggest the path I should look at for the next week, and seeing him the following Monday. Lather, rinse, and repeat. Now I was on my own. To make a long story short, I decided I wanted to move away from my thesis (but still do research in real analysis), and so I started looking through journals for something that interested me. What I did not realize was that my thesis was not just guided research, but pretty heavily-guided research. Working on my own was going to be hard.

It's been long enough that I don't remember the topic, but I do remember trying to find some results and thinking I had some small, but clever ideas. In my naive state, I thought that even a small thing, whether a positive proof or a negative example, would be interesting and publishable. I was one of the first in my department to get a desktop computer and one of only two who knew LaTeX and could typeset mathematics to look nice. So, I typed up my results and sent them to my old advisor so that I could get some nice kudos that would keep me going. A quick reply was not forthcoming. Finally, he wrote back, and the subject line said it all: "Ugh." I'll skip the sordid details. I thought I had results. He said they were obvious, not interesting, and not research. But professional development at my institution was the name of the game, and I thought about taking myself in lots of other directions, giving up research and sticking with things I knew I could do. So, what was my next move? I moped. Yeah, I felt sorry for myself. Then I got better. Then I got mad.

And now I had a goal to focus on. An angry goal, but still a goal: to show to my advisor (and others) that I could get something done.

I started by reading lots of journal articles. Specifically, I was looking for articles with open problems in them. This way, I could get my hands on some problems rather than determine my own, and I could start to learn what are good questions. I got some decent results about the metric space of metrics. Boom! Paper. I realized there's no harm in asking others, so at a meeting, I asked an older friend if he had anything we could work on together. He did, and that was another paper. Boom, again! Then, I lucked out. I found a paper on metric-preserving functions that had an open question, and one of the authors was at a school about an hour away from me. I emailed to ask if the question was still open, and it was. It took me over a year, but I answered it with a really nice, deep counterexample. Getting to know this person gave me the opportunity to (a) go to his school to both attend and present at their mathematics colloquium, (b) have someone to talk with, and (c) have a

place to go to when I took a sabbatical. Let me point out that this took years. It was a marathon, not a foot race. It's ongoing today.

So, I learned how to ask and answer questions. Sometimes, I'm the only one interested in them. I give talks, and nobody has anything to say, but that's OK. I think they're good. I've moved on from my first tenure-track job, but for the decades I was there, I averaged a research paper a year and have branched out into many topics (analysis, topology, number theory, recreational mathematics, etc.). I have also written expository and pedagogical papers and a book. My perseverance paid off. I'm sure that yours will, too.

Robert Vallin earned his PhD in real analysis from North Carolina State University in 1991. During the last decade he turned his attention to recreational mathematics, producing research on the mathematics of KenKen puzzles, games, and magic tricks. He is the author of a book on Cantor sets, two sets of class notes in real analysis, and over 40 articles on mathematical research, pedagogy, and exposition. He is currently a member of the mathematics department at Lamar University in Beaumont, TX. When not doing math he is spending time with his wife, Jackie, and a subset of his five children, practicing close-up magic, or falling asleep with his cat on him.

39

Anxiety Attacked Me, But I Survived

Ken Millett

Courtesy of American Mathematical Society

Coming from Oconomowoc High School, my undergraduate experience as a wannabe engineer at MIT was marked by some serious obstacles. I was not prepared for life in a large city, not aware of MIT's reputation or its intense academic environment, nor was I prepared for the intellectual level and preparation of my classmates. My parents drove me to my cooperative residence in Boston. There, I was greeted by classmates who questioned how many 800s I had on the SATs. I was paralyzed. Not only did I not imagine that a person could get an 800, but I was astounded that the measure of success was "how many." It was clearly impossible to get back into my parents' car and say, "Take me home. I've made a big mistake." It was like being on another planet with no hope of escape.

As I progressed in the pursuit of my bachelor's degree, I continued to be plagued by a feeling of inadequacy which had one highly unfortunate effect: it made taking exams nearly impossible. One of my more memorable experiences was a three-hour exam during which I only managed to put my name on the paper and sit terrorized by a giant ticking clock on the wall

for the entire exam period. Fortunately, MIT was stronger than me and did not let me give up. With the encouragement of housemates and with many months of counseling and work, I learned enough to survive examinations, though often barely. I learned to have a meta-awareness of situations I'd find myself in—for example, becoming paralyzed during an exam. I learned how to stop, step back, breathe deeply, close my eyes, and imagine myself in a favorite place—such as hiking the peaks in New Hampshire's White Mountains—until I calmed down and was able to restart work on a problem.

While I was working on a construction crew during one hot, humid summer to pay for tuition, I resolved to return to MIT and to ask all my dumb questions so that my professors would have to help even dumb students like me. Of course, I eventually learned that my classmates had the same questions that I did, and—bless their hearts—the faculty and staff never seemed to give up on me. They gracefully answered every one of my questions and more. For example, from my advisor Dan Kan (with whom I had to meet when I proposed to leave MIT for a less demanding school), I learned how to organize my efforts to optimize my learning. For example, he taught me: "Don't try to learn when too tired." He proposed a daily schedule that included a mere ten hours per day of school work, eight hours of sleep, time for meals and, believe it or not, time for "fun." I tried this for a month or so before final exams one semester in my second year, and I had the most surprising experience. I had more fun, and my grades in EVERY CLASS increased by at least a letter. This advice, with small refinements, still informs my daily schedule.

Nevertheless, I still managed to create occasions for MIT to work hard to help me overcome my weaknesses. Exam anxiety was never entirely conquered and, as a consequence, I almost didn't graduate with my class in 1963. As a senior math major, I had to complete all my courses before graduation in June and get a job to support my family. The pressure and anxiety increasingly overwhelmed me to the point that I was struggling quite a bit to get *anything* done. In the end, I got confused about when my last final exam would be held. I showed up, but nobody was in the room at the appointed time, and I had to find my professor to find out when the exam was going to happen. Alas, it was the day before! Fortunately, MIT and my professor, Isadore Singer, were up to the challenge of helping me graduate. Professor Singer got MIT to allow me to take an oral exam in order to pass the course and graduate with my class. I have been—and continue to be—deeply grateful to MIT and its faculty for all that I learned. They made it possible for me to prepare for a career in mathematics and support my family, despite all the

trouble I had along the way. As an undergraduate, mathematics was never easy for me, but it was really, really interesting.

After graduation, I was offered an excellent job with a generous salary at one of the military research support companies. I was assigned to a team working on developing and testing communication and sensing systems in support of governments funded by the U.S. After only a few months, I realized that the work I was doing was in conflict with my beliefs and ethical principles. Despite needing to support my family, I had to resign. Fortunately, with the assistance of my parents and letters of recommendation from my MIT professors, I was able to return to Wisconsin where the mathematics department was willing to admit me to their graduate program, albeit on probation. They gave me a chance to earn a master's degree and find a job that was more compatible with my pacifist beliefs and ethical principles. To UW-Madison and its faculty and my MIT recommenders I owe this opportunity to enter a program where, finally, I was successful.

The expectations of a doctoral program, I learned, consisted of things I could do and that I really enjoyed. The depth of understanding expected, the daily adventure of learning amazing things, and the fellowship of so many other students who were similarly confronting confusion made my anxiety seem quite normal and well within a range that I could conquer. From my research advisor, Edward Fadell, I learned so much more about doing mathematics, teaching, and working with students. It was as a graduate student in 1964 when I started teaching that I first recognized myself as a "mathematician." This made it clear to me that I was on the right path. Now, more than 50 years later, I look back to MIT and UW-Madison and their faculties and feel very grateful for all that they did to allow me to have the opportunity to become a mathematics teacher (my job) and researcher (my pleasure) and give back to our mathematics community for all that I have received.

This "giving back" has been an important part of my life. After joining UCSB, my first years were focused on my own professional growth, but I soon began working with minority STEM students. With the help of the Equal Opportunity Program staff, I created and found funding for a new, two-week summer residential program for 15 female and 15 male admitted STEM students who were first generation in college, from underrepresented minority groups, or from socioeconomically disadvantaged communities. My dream was for these students to be prepared for success, to find friends like themselves, and to have a support system of graduate students, staff, and faculty to whom they could turn for advice. The experience was amazing. These students became campus leaders. They not only graduated—a rare

outcome for such students back then—but many went on to graduate school and found their homes in STEM professions. This program, the Summer Institute in Mathematics and Science (SIMS), continues under a new generation of leadership and with ongoing campus funding. Over the years, I continued to try give back for all that I had received. I served as the regional director of the National Science Foundation (NSF) supported California Alliance for Minority Participation (CAMP), and I created the California Mathematics and Science Teacher (CMST) program.

Thinking back, I feel I should share an important regret relevant to these efforts. At some point, I was directing three very intense programs as well as supervising PhD students. I felt that I did not have enough energy to continue work on these three programs while having my own research be supported by the NSF. I decided to reduce my stress level by no longer seeking research grants from the NSF. I was not actually reducing my research effort, but at least I didn't have to question whether my research was of high enough quality to be funded by the NSF. I was wrong to not request NSF funding for two reasons. First, and very importantly, I eliminated the opportunity to financially assist the students and postdocs who were working with me and my colleagues. Second, I should have left the funding decision to the NSF and its review panels; they might not have had the same opinion of my productivity as I did. And what's the worst thing that could have happened? My research wouldn't have been funded, and I'd be right back where I started. I am reminded of my father's admonition: "If you are not failing every once in a while, you are just not trying hard enough." And, of course, there was my grandmother, who would tell me, "If you want to dance with a 'pretty girl,' you have to ask her." Even though I was twelve at the time and her advice was given in another context, her words continue to ring in my ears when I consider my choices in life.

Kenneth Millett is an emeritus professor at the University of California, Santa Barbara whose faculty he joined in 1964 following a lectureship at MIT, 1967-1969. After growing up in Oconomowoc, Wisconsin, he completed his undergraduate degree at MIT, 1959-1963. He later earned his MS, in 1964, and PhD, in 1967, from the University of Wisconsin, Madison. His research interests span areas of geometry and topology of 3-space structures, especially knot theory with applications to molecular biology, biochemistry, biophysics, polymer physics, and fluids. Mathematically, these include the creation of knot invariants (HOMFLYPT polynomial) and the mathematical analysis of knotting, slipknotting, and linking in proteins and polymer melts. He has been dedi-

cated to increasing the participation and success of STEM underrepresented individuals with service across a broad spectrum of organizations such as the College Board, AAAS, SACNAS, AMS, MAA, and the University of California and in a variety of roles. He is a fellow of the AMS and AAAS and has been recognized with the Allendoerfer Award, the Chauvenet Prize, and the AMS Award for Distinguished Public Service.

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It's Like a Jungle Sometimes, It Makes Me Wonder How I Keep From Going Under

Christina Eubanks-Turner

Photo taken by Earl Turner III

One piece of advice I would give my younger self is to tell her that she WILL accomplish everything she puts her mind to with patience and time. My time as a graduate student truly validated this for me. As an African-American female and a proud graduate of a historically black college or university (HBCU), I was sure that when I decided to pursue a doctoral degree in mathematics, I would also attend an HBCU for graduate studies. I could not have been more wrong. After visiting several schools, my family and I decided on a large predominately-white institution (PWI) in the Midwest with a department that had a high percentage of women but lacked ethnic and cultural diversity. While attending graduate school at a large, midwestern PWI intimidated me, I had another task that concerned me more.

A year before I was to start graduate school, I had given birth to my first daughter. I had already struggled to complete my senior year as an undergraduate with a child, but now my fiancé and I had to contemplate what would be best for our family. Should we stay close to home and enter the

workforce to make money to support our new family? Or should we leave so that I could pursue a graduate degree and accept a lower-paying teaching assistantship while securing a future career? As I weighed my options, the fact that I could potentially be one of the first persons of color to gain a PhD in math from my graduate institution played a role in my decision-making. In the end, I decided to further my education.

After getting married a month before I left for graduate school, I had a lot of anxiety about being a new wife, mother, and graduate student in a city where I had no support from family and friends. I was thankful for the opportunities I had been afforded, but I still felt very nervous about the changes that were taking place in my life. Upon arriving in the small town where I would live for the next several years of my life, I remember saying to myself, "What have I done?" I felt like a fish out of water, and although the faculty, staff, and my peers in the math department were friendly, I remember going into stores where the cashier dropped my change on the counter and did not place it in my hand. I recall my husband telling me of an instance where he was walking down the street and a group of men in a truck passed and called him the N-word. I felt invisibly noticed, that is, I was invisible as no one spoke to me, yet they all stared.

In my department, I soon learned that I was the only graduate student in my cohort with a child. As my husband searched for employment, we found it hard as a family of three to get by on a teaching assistantship paying an annual salary of around $15,000. We tried for government assistance, but they stated that I was not eligible because I was an employed graduate student, no matter how low my income was. During these times, I was full of self-doubt, and I wanted to give up. To top it off, I was struggling to understand the concepts in my courses and falling behind with my work. I also served as a recitation instructor for the first time and specifically remember numerous white males in my courses questioning my mathematical abilities, forcing me to engage in "mathematical showdowns," to validate that I knew enough mathematics to teach the course. Once I did this and the student essentially "lost," I was able to gain control of the classroom and earn some kind of trust from my students that I was a capable instructor.

If the stress and pressure from graduate school weren't enough, the pressures from home at times were just as heavy. With a daughter in her "terrible-twos" and a husband who was newly employed, I look back to those times and wonder how we remained sane. I definitely remember having arguments with my husband that stemmed from financial concerns and how much time I was spending at the office. At this point, we did not have a

strong support system, and I was too embarrassed to admit that I needed help. Mentally, I was about to crack. Fortunately, at a point when I needed it the most, I had a realization: I was not the typical graduate student, and I should embrace that fact. Instead of trying to fit the mold and lifestyle of a typical graduate student, I should boldly stand out.

Although I needed to study as much as, if not more than, the others in my department, when I was at home, my role as wife and mother commanded my attention. So, I had to learn that sometimes studying off campus by myself was the only way to get work done. This was far from ideal, since I benefited tremendously from studying with peers, but it helped me stay on track. Also, when my daughter was able to play quietly by herself, I started bringing her to study sessions. To my surprise, some of my peers enjoyed having her there, as they liked the intermittent distraction. Around the same time, my husband became more visible in the department as I invited him to more of the department events. Ultimately, I ditched the self-guilt and realized that there is no perfect mom, wife, or student. I am fine with being imperfect and learning along the way.

As my husband and I reminisce on those times, we laugh, cry, and wonder how we made it through. We realize that those times laid the foundation for our beautiful, strong 17-year marriage. I look at my amazing daughters—one of whom is preparing to go to college, and the other, a fourth-grader full of curiosity—and think of how blessed we are. Although I started my family before my career, which was very hard, I look at my colleagues who are starting families in the midst of their careers. Seeing some of the issues they have to deal with (tenure clocks, maternity/paternity leave), I realize there is no perfect time to start a family. I know now that you have to just take things as they come and believe in yourself.

Christina Eubanks-Turner is an associate professor of mathematics at Loyola Marymount University in Los Angeles, CA. She received her BS in mathematics from Xavier University of Louisiana in New Orleans, LA and both her MS and PhD in mathematics from the University of Nebraska-Lincoln in 2008. Her current research interests are in commutative algebra with an emphasis in ring and module theory, graph theory, math education and broadening participation in the field of mathematics.

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Should I Quit Mathematics?

Francis Su

"Should I quit mathematics?" This was a difficult question for me to answer for all sorts of reasons, most of which had very little to do with mathematics. I was in my fourth year of my PhD program. I had already been a disappointing student to two PhD advisors—one who was mostly absent for the two years I worked with him, and the second who was difficult to please and quite unkind, letting me know that a "real mathematician" wouldn't do the things he felt I was doing. He was implying, of course, that I didn't belong in that category.

I had come to graduate school less well-prepared than my peers, and I had a rough transition, in part because my mother was dying of ALS and I had a hard time focusing on my work. I was also meeting, for the first time, people who were supremely talented mathematically, and I felt I didn't measure up. Moreover, I wanted more life balance than I witnessed my professors having. All these factors contributed to a crisis of self-doubt that was slowly stealing my joy.

I had dreamed of becoming a mathematician, but I also did not want to continue in a day-to-day reality that felt oppressive. So, three-and-a-half years into a PhD program, I began asking whether I should leave and do something else.

If you're asking this question like I did, I want to affirm that asking the question is not a sign of weakness, but a sign of strength. You are taking control of a situation that has, until now, felt out of your control. Each of us who ask the question may ultimately answer it in different ways, but I want to share some of the good things that came out of wrestling with the question.

First, I realized that my dignity did not have to come from getting this PhD. Somehow, I had made it the ultimate marker of whether I was a worthy human being. And yet, when I reflected on my experiences, I saw instead that my most joyful moments of feeling loved and accepted came from people who didn't care whether I got a PhD. I also began to imagine other careers that I might pursue, and just doing that exercise felt freeing.

Second, I realized that comparing myself to other people was always going to be self-defeating, no matter what career I pursued. There will always be someone more skilled at things I feel I am skilled at. Becoming more centered in who I was (e.g., the things I love and why I love them), rather than what I could accomplish, has helped to free me from the pressure to compare myself to others.

Third, I realized that graduate school can give you a pretty skewed view of what being a mathematician is like. You are training at a research university where research is prized above all else. But there are many other important facets of being a scholarly mathematician: being an excellent teacher and communicator and mentor of mathematics is, I believe, equally important. In speaking with mathematicians outside the research bubble, I saw that it was possible to have a life balance that doesn't prioritize research over all else.

Now, having gone through this soul-searching has brought benefits. When I experience failure, it rattles me less than it used to. When I experience success, I don't cling to it too tightly. Because I know it's not where I get my dignity.

I'm also more willing to take risks—like sharing with you all that I have self-doubts and admitting that I have weaknesses—because I don't get my dignity from appearing successful or talented or having it all together. I also know that success or failure in a worldly sense has very little to do with talent, and that worldly success is truly not as important as the kind of human beings we are to one another.

In graduate school, I was fortunate when another professor suggested that I work with him. He had little reason to believe that I was going to be a successful student, but he took me as his student anyway. He exemplified what a true advisor should be, and the ways we need to be supporting each other.

But I also was realistic about this opportunity, and I gave myself a time limit. If at the end of a preset time period, I was making positive progress towards my PhD, I would stay and continue—and that's ultimately what happened. However, I also knew that, even if it didn't work out, I would do something else, and that alternative could be just fine. I could still love mathematics for its own sake while being involved in some other exciting career possibility.

In your own path, whatever happens, you can thrive. Nevertheless, I do want to say that despite what anyone may tell you, there is a place for you in the mathematics community that may not be visible yet from your vantage point. You should never have to leave because someone's discouraged you. If you do, that burden lies on the mathematics profession. We can and we must do better to welcome all those who love mathematics.

Francis Su is the Benediktsson-Karwa Professor of Mathematics at Harvey Mudd College. In 2015 and 2016, he served as President of the MAA. His research is in geometric combinatorics and applications to the social sciences, and many of his papers have been co-authored by undergraduates. From the MAA, he received the 2018 Halmos-Ford award for distinguished writing, and the 2013 Haimo Award for exemplary teaching. His Haimo lecture "The Lesson of Grace in Teaching," which described more of his graduate school struggles, appeared in the Princeton anthology The Best Writing on Mathematics 2014*. His speeches and public writing have sought to describe the humanity of mathematics and call people to greater awareness of issues that contribute to inequitable mathematics education. His book* Mathematics for Human Flourishing *will be published by Yale University Press in 2019.*

LIVING PROOF

STORIES OF RESILIENCE ALONG THE MATHEMATICAL JOURNEY

EDITED BY:
ALLISON K. HENRICH
EMILLE D. LAWRENCE
MATTHEW A. PONS
DAVID G. TAYLOR

WOW! This is a powerful book that addresses a long-standing elephant in the mathematics room. Many people learning math ask "Why is math so hard for me while everyone else understands it?" and "Am I good enough to succeed in math?" In answering these questions the book shares personal stories from many now-accomplished mathematicians affirming that "You are not alone; math is hard for everyone" and "Yes; you are good enough." Along the way the book addresses other issues such as biases and prejudices that mathematicians encounter, and it provides inspiration and emotional support for mathematicians ranging from the experienced professor to the struggling mathematics student.

– Michael Dorff, MAA President

THIS BOOK is a remarkable collection of personal reflections on what it means to be, and to become, a mathematician. Each story reveals a unique and refreshing understanding of the barriers erected by our cultural focus on "math is hard". Indeed, mathematics is hard, and so are many other things - as Stephen Kennedy points out in his cogent introduction. This collection of essays offers inspiration to students of mathematics and to mathematicians at every career stage.

– Jill Pipher, AMS President

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