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Michael S Lang* (mlang@math.wisc.edu). *Bipartite Distance-Regular Graphs, Three-Term Recurrent Eigenvalues, and Representation Diagrams.*

Let Γ denote a bipartite distance-regular graph with diameter $D \geq 4$ and valency $k \geq 3$. Let θ denote an eigenvalue of Γ other than $k, -k$ and let $\sigma_0, \sigma_1, \dots, \sigma_D$ denote the associated cosine sequence. We show

$$(\sigma_1 - \sigma_{i+1})(\sigma_1 - \sigma_{i-1}) \geq (\sigma_2 - \sigma_i)(\sigma_0 - \sigma_i)$$

for $1 \leq i \leq D - 1$. We show the following are equivalent: (i) equality is attained above for $i = 3$ (ii) equality is attained above for $1 \leq i \leq D - 1$ (iii) the cosines obey a linear three-term recurrence. We say θ is three-term recurrent (or TTR) whenever (i)–(iii) are satisfied. We relate TTR eigenvalues to the Q -polynomial property. When an eigenvalue is TTR, we find formulae for the intersection numbers and eigenvalues of Γ in terms of two parameters, classifying Γ in some cases. Among the eigenvalues in their natural order, we consider which can be TTR. If Γ has more than one TTR eigenvalue, we show Γ is either the D -cube or antipodal with $D \leq 5$. Let Δ denote the θ -representation diagram. For $D > 6$, we show the following are equivalent: (a) in Δ , θ is adjacent to at most one vertex other than k (b) Δ is either a path or two paths (c) θ is TTR. (Received September 14, 2000)