

962-11-735

**Donald Mills\*** (ad3943@usma.edu), Department of Mathematical Sciences, West Point, NY 10996, and **Gavin McNay** (gavin.mcnay@nomura.co.uk), 20 Siddons Road, Tottenham, N17 London, England. *Primitive Roots in Cubic and Higher Extensions of a Finite Field.*

In 1983 S.D. Cohen proved that for any finite field  $GF(q)$  and for any element  $\theta$  such that  $GF(q)(\theta) = GF(q^2)$  there exist elements  $a, b \in GF(q)$  such that  $a\theta + b$  is a primitive root of  $GF(q^2)$ . We consider the same question, but for higher extensions, namely cubic, quartic, and quintic extensions. We prove that for any finite field  $GF(q)$  and for any element  $\theta$  such that  $GF(q)(\theta) = GF(q^3)$  there exist elements  $a, b \in GF(q)$  such that  $a\theta + b$  is a primitive root of  $GF(q^3)$ . We give asymptotic results for the quartic and quintic cases as well. Additionally, we address the more difficult question of whether the above problem can be resolved in the affirmative when  $a = 1$  and  $\theta$  is a defining element of either  $GF(q^3)$  or  $GF(q^4)$ . These primitive sums can be used as pseudo-random vector generators; we illustrate this application for  $n = 3$ . (Received September 24, 2000)