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**Carl Pomerance\*** ([carlp@lucent.com](mailto:carlp@lucent.com)), Bell Labs - Lucent Technologies, 600 Mountain Avenue, Murray Hill, NJ 07974. *Towards an Artin conjecture for composites*. Preliminary report.

The strong form of Artin's famous conjecture on primitive roots is that if integer  $a$  is not  $-1$  nor a square, then there is a positive proportion of prime numbers which have  $a$  as primitive root. Hooley has proved this conditional on the GRH. We export the concept of primitive root to composite moduli as follows:  $a$  is a "primitive root" for  $n$  if  $a$  is coprime to  $n$  and the multiplicative order of  $a$  modulo  $n$  is the maximum of all multiplicative orders modulo  $n$ . Let  $\mathcal{R}(a)$  denote the set of such numbers  $n$ . It is tempting to conjecture that if  $a$  lies outside some small exceptional set, then the set  $\mathcal{R}(a)$  has positive asymptotic density. However, Shuguang Li has shown this is not true: each set  $\mathcal{R}(a)$  has lower density 0. Li conjectured though that for  $a$  outside a small and explicit exceptional set,  $\mathcal{R}(a)$  has positive upper density. In this talk I discuss an approach for Li's conjecture that is conditional on the GRH. (Received September 26, 2000)