

962-11-818

**Pantelimon Stanica\*** ([stanpan@strudel.aum.edu](mailto:stanpan@strudel.aum.edu)), Department of Mathematics, Auburn University Montgomery, Montgomery, AL 36117. *Divisibility of Binomial Coefficients and Catalan Numbers.*

Problems involving binomial coefficients were considered by many mathematicians for over two centuries. R.K. Guy in his "Unsolved Problems in Number Theory" (B31,B33, etc.), mentions several problems on divisibility of binomial coefficients. One famous example is that of the Catalan numbers  $[1/(n+1)] \text{binom}\{2n, n\}$ . Erdos conjectured that for  $n > 4$ ,  $\text{binom}\{2n, n\}$  is never squarefree. This was proved by Sarkozy, for sufficiently large  $n$ , and by Granville and Ramare for any  $n > 4$ . Erdos, Graham, Rusza and Straus showed that for any primes  $p, q$  there are infinitely many  $n$  for which  $\text{gcd}(\text{binom}\{2n, n\}, pq) = 1$ . Hough and the late Simion propose (although they are not the first) the following generalization, which we will call s-Catalan numbers,  $F(s, n) = [1/((s-1)n+1)] \text{binom}\{sn, n\}$  and naturally the questions that arise are: (a) When  $p$  is prime, for what values of  $n$  is  $F(p, n)$  divisible by  $p$ ? (b)\* For what values of  $n$  is  $F(4, n)$  divisible by 4? (c)\* What can you say when  $s$  takes on the other composite values? There are no known answers for (b),(c). We prove (b) and partially (c). We also give some results on the divisibility of binomial coefficients using a method of Granville and Ramare, and bounds on exponential sums. (Received September 26, 2000)