

962-13-1132

**Thomas G. Lucas\*** (tglucas@email.uncc.edu), Department of Mathematics, University of North Carolina Charlotte, Charlotte, NC 28223. *Rings, conditional expectations and localizations*. Preliminary report.

Let  $R \subset S$  be a pair of reduced rings with the same identity where the total quotient ring of each is von Neumann regular. Let  $E_R$  ( $E_S$ ) denote the set of idempotents of  $R$  ( $S$ ) and assume that each element  $r \in R$  ( $s \in S$ ) can be written in the form  $r = et$  ( $s = fv$ ) for some regular element  $t \in R$  ( $v \in S$ ) and some idempotent  $e \in E_R$  ( $f \in E_S$ ). For a pair of idempotents  $e$  and  $f$ , set  $e \leq f$  if  $ef = e$ . An  $R$ -module homomorphism  $\varphi : S \rightarrow R$  is said to be a “conditional expectation” if (i) for  $f \in E_S$ ,  $\varphi(f) = 0$  implies  $f = 0$ , and (ii) if  $\varphi(sf) = 0$  for each  $f \in E_S$ , then  $s = 0$ . Assume such a mapping exists. Then for each  $f \in E_S$  there is a unique pair of idempotents  $f^\sharp, f_\sharp \in E_R$  such that (i)  $f_\sharp \leq f \leq f^\sharp$ , (ii)  $g \in E_R$  with  $f \leq g$  implies  $f^\sharp \leq g$ , and (iii)  $h \in E_R$  with  $h \leq f$  implies  $h \leq f_\sharp$ . Fix  $f \in E_S$  and set  $t = \varphi(f) + (1 - f^\sharp)$ . Let  $T = \{t^n | n \geq 0\}$  and say that  $f$  “localizes”  $R$  if  $fS_T = fR_T$ . Several equivalent conditions will be given. Examples will be drawn from rings of  $L^\infty$  functions of comparable complete probability measures. (Received October 02, 2000)