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A commutative ring  $R$  is condensed (strongly condensed) if for ideals  $I, J$ ,  $IJ = \{ij | i \in I, j \in J\}$  ( $IJ = iJ$  for some  $i \in I$  or  $IJ = Ij$  for some  $j \in J$ ). Condensed domains were introduced by D.F. Anderson and Dobbs, strongly condensed domains by Gottlieb. We show that for a Noetherian domain  $D$ ,  $D$  is condensed  $\iff \text{Pic}(D) = 0$  and  $D$  is locally condensed, while  $D$  is strongly condensed  $\iff D$  is a PID or  $D$  has exactly one maximal ideal  $M$  that is not principal and  $D_M$  is strongly condensed  $\iff \dim D \leq 1$ ,  $\text{Pic}(D) = 0$ , and  $D'/D$  is serial. A domain  $D$  is strongly condensed  $\iff D$  is a PID or  $D$  has exactly one maximal ideal  $M$  not principal of height one,  $D_M$  is strongly condensed, and  $D$  has Noetherian spectrum. An integrally closed domain  $D$  is strongly condensed  $\iff D$  is a Bezout generalized Dedekind domain with at most one maximal ideal of height greater than one. A local domain is strongly condensed  $\iff$  it has the two-generator property. We give equivalencies for a local domain with finite integral closure to be strongly condensed. For fields  $k \subseteq K$ , the domain  $D = k + XK[[X]]$  is condensed  $\iff [K : k] \leq 2$  or  $[K : k] = 3$  and each degree-two polynomial in  $k[X]$  splits over  $k$ , but  $D$  is strongly condensed  $\iff [K : k] \leq 2$ . (Received September 07, 2000)