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On the Cale Property in Integral Domains and Monoids.

A commutative cancellative monoid M with unit 1 is a Cale monoid with base $Q \subseteq M$ if for each nonunit $x \in M$ there is a factorization $x^{m(x)} = u \prod_{q \in Q} q^{x(q)}$ where u is a unit and for $m(x) \in \mathbb{N}$ minimal the exponents $x(q) \in \mathbb{N}_0$ are uniquely determined ($x(q) \neq 0$ only for finitely many q). An integral domain is a Cale domain if its multiplicative semigroup is a Cale monoid. The Cale property provides a generalized kind of unique factorization which holds for a wide variety of monoids and domains. Examples range from Krull domains and Diophantine monoids to, not necessarily integrally closed, polynomial rings, orders of algebraic number fields and numerical semigroups in higher dimensions. A principal result states that monoids/domains whose root-closure is Krull with torsion divisor class group, can be internally characterized as Cale monoids/domains satisfying an integrality condition. One then obtains in particular the following characterizations. A Diophantine monoid is a Cale monoid iff its defining integer-valued matrix satisfies some rank condition. An arbitrary order of an algebraic number field is a Cale domain iff powers of elements obey some simple divisibility rule. (Received September 25, 2000)