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60614. *On the second quantum derivative*. Preliminary report.

The second quantum derivative  $\mathcal{D}_2f(x)$  of the real-valued function  $f$  is defined for each real non-zero  $x$  as

$$\lim_{q \rightarrow 1} \frac{\frac{f(q^2x) - f(qx)}{q^2x - qx} - \frac{f(qx) - f(x)}{qx - x}}{qx - x}.$$

If the second Peano derivative exists at  $x$ , which is to say that if  $f$  can be approximated by a quadratic polynomial at the point  $x$ , then it is easy to see that  $\mathcal{D}_2f(x)$  must also exist at that point. Consideration of the function  $|1 - x|$  at  $x = 1$  shows that the second quantum derivative is more general than the second Peano derivative. However, we can show that the existence of the second quantum derivative at each point of a set necessarily implies the existence of the second Peano derivative at almost every point of that set. We will also discuss higher order quantum derivatives and possibly further generalizations of the differentiation process. (Received September 20, 2000)