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Massachusetts Ave. NW, Washington, DC 20016-8050. *Counting Functional Roots*. Preliminary  
report.

The set of complex valued functions analytic in a neighborhood of zero and vanishing at zero is closed under composition. We consider the existence of functional roots in this setting. A function  $g$  is a functional  $n^{\text{th}}$  root of  $f$  for  $n > 1$  if the  $n^{\text{th}}$  iterate of  $g$  equals  $f$  in a neighborhood of zero. Examples are given of functions which have: (i) no  $n^{\text{th}}$  roots for any  $n$ ; (ii) infinitely many  $n^{\text{th}}$  roots for each  $n$ ; (iii) exactly  $n$   $n^{\text{th}}$  roots for each  $n$ ; (iv) exactly one  $n^{\text{th}}$  root for each  $n$ ; (v) exactly two square roots, three cube roots, six  $6^{\text{th}}$  roots, and one  $n^{\text{th}}$  root for every other  $n$ ; (vi) exactly nine square roots, 21 cube roots, 63  $6^{\text{th}}$  roots, and no  $n^{\text{th}}$  roots for any other  $n$ . The number of roots which a function has is related to the set of functions which commute with that function. We will discuss classes of functions for which the above examples are typical. (Received September 14, 2000)