962-34-642 **Kwang C Shin*** (kcshin@math.uiuc.edu), Department of Mathematics, University of Illinois, 1409 W. Green Street, Urbana, IL 61801. *On the Eigenproblems of $\mathcal{PT}$-Symmetric Oscillators.*

We consider the ordinary differential operators $H_n = -\frac{d^2}{dx^2} + P(x^2) - (ix)^{2n+1}$ in $L^2(\mathbb{R})$ where $n \geq 1$ and P is a real polynomial of degree at most n. In 1995, D. Bessis and others conjectured that eigenvalues of the Hamilitonian $H_1 = -\frac{d^2}{dx^2} + ix^3$ (with $P \equiv 0$) are all real and positive. Many numerical and asymptotic results support this conjecture. However, there is no rigorous proof of this to date. In this talk, we will prove that if eigenvalues λ of H_n exist, then they lie in the sector $|\arg \lambda| < \frac{\pi}{2n+3}$. Also for the case H_1, we establish a large zero-free region of the eigenfunctions and their first derivatives. (Received September 18, 2000)