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Canan Celik* (celikcan@math.msu.edu), Department of Mathematics, Michigan State University, East Lansing, MI 48824. *No Local L^1 Solution for a Nonlinear Heat Equation.*
Preliminary report.

In this paper we consider the nonlinear heat equation $u_t = u_{xx} + |u|^{p-1}u$ on $\mathbb{R}^+ \times (-1, 1)$ with vanishing dirichlet boundary condition and the initial condition $u(x, 0) = u_0(x)$ in $L^1(-1, 1)$, where $p > 1$. It has been well known for $p < 3$ that this problem has a local solution for any initial condition $u_0 \in L^1(-1, 1)$. But the existence and uniqueness of the local solution in L^1 for the critical exponent $p = 3$ was widely open and this work is to answer to this open problem. By using a delicate dilation argument, we first prove the finite time blow-up of the solution for a particular explicit initial data u_0 for the critical exponent $p = 3$, which is been used to construct a class of initial data $u_0 \in L^1(-1, 1)$ for which there is no local L^1 solution. We also establish the global existence in $L^{1+\epsilon}$ with $\|u_0\|_{1+\epsilon}$ sufficiently small and $\epsilon > 0$ for the critical exponent $p = 3$. (Received October 03, 2000)