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The equation  $f(x + y) - f(x) - f(y) = u(x,y)$  is what I call the nonhomogeneous linear Cauchy functional equation, by analogy with ordinary differential equations. The function  $u(x,y)$  can be thought of as the control or driving function for  $f(x)$ , again by analogy with ordinary differential equation control problems. Obviously, given  $f$ ,  $u$  is determined uniquely. However, given  $u$ ,  $f$  may not exist, or if it does, it may not be unique. These equations are useful in combinatorics in describing sums of the powers of integers and combinations with various repetitions, and in some applications in economics. Questions that arise regarding the equation are: (1) What are necessary and/or sufficient conditions on  $u$  for an  $f$  to exist?; (2) Given such a  $u$ , how do you find  $f$ ?; (3) With additional requirements on  $f$ , such as that it must satisfy given endpoint conditions, are there any  $u$ 's that will work?; and (4) If more than one  $u$  will drive the system as required, what additional requirements can be imposed on  $u$ , such as optimizing some function of  $u$  and/or  $f$ ? I have developed several necessary conditions for  $u$  and recently a student of mine, Steven K. Butler, showed that with sufficient smoothness assumptions they are sufficient. These necessary and sufficient conditions will be discussed. (Received October 04, 2000)