

962-52-1391

**Dan P Ismailescu\*** (ismailes@cims.nyu.edu), Courant Institute of Mathematical Sciences,  
New York University, 251 Mercer Street, New York City, NY 10012. *On a question of Erdős.*

Given  $k \geq 3$  and a set  $A_n$  of  $n$  points in the plane, we shall denote by  $t_k(A_n)$  the number of lines containing precisely  $k$  of the points. Erdős (1962) raised the following problem: how large  $t_k(A_n)$  can be, given that there are no  $(k + 1)$  collinear points? Let  $L_k(n)$  denote the maximum of  $t_k(A_n)$  when  $A_n$  varies over all sets of  $n$  points in the plane that contain no collinear  $(k + 1)$ -tuple. Most of the results obtained so far are for the particular case  $k = 3$  (also known as “the orchard problem”). Much less is known for  $k \geq 4$ . Grünbaum (1976) proved that

$$L_k(n) \geq c_k n^{(k-1)/(k-2)}.$$

We present a different construction which implies that

$$L_k(n) \geq c'_k n^{\log(k+4)/\log(k)}.$$

This matches Grünbaum’s bound for  $k = 4$  and it is strictly better for all  $5 \leq k \leq 35$ . (Received October 03, 2000)