962-52-741

Jeffrey C. Lagarias* (jcl@research.att.com), Room C235, AT&T Labs, 180 Park Avenue, Building 103, Florham Park, NJ 07932-0971, **Colin L Mallows** (clm@research.att.com), Room C285, AT&T Labs, 180 Park Avenue, Building 103, Florham Park, NJ 07932-0971, and **Allan R Wilks** (allan@research.att.com), Room C207, AT&T Labs, 180 Park Avenue, Building 103, Florham Park, NJ 07932-0971. *Beyond the Descartes Circle Theorem.*

The Descartes circle theorem states that if four circles in the plane are mutually tangent and have disjoint interiors, then their curvatures (or "bends") $b_i = 1/r_i$ satisfy the relation $(b_1 + b_2 + b_3 + b_4)^2 = 2(b_1^2 + b_2^2 + b_3^2 + b_4^2)$. We show that similar relations hold involving the centers of four circles in such a configuration, coordinatized as complex numbers, which we call the complex Descartes theorem. Futhermore these relations have matrix generalizations to the n-dimensional case, in each of Euclidean, spherical and hyperbolic n-space. In the process we obtain direct analogues of the Descartes circle theorem valid in spherical and hyperbolic n-space. These results led to a large project studying Apollonian circle packings and their n-dimensional generalizations, with associated group theory and number theory questions, which is joint work of the authors with R. L. Graham (UCSD) and C. Yan (Texas A & M). (Received September 24, 2000)