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Linda E. McGuire* (lmcguire@muhlenberg.edu), Department of Mathematical Sciences,
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Given any graph, we can easily determine the degree of each of its vertices and present this information in the form of a nonincreasing sequence of nonnegative integers. This sequence is called the *degree sequence* of the graph.

Can this process be reversed? Specifically, given a sequence S of nonnegative integers, does it represent the degree sequence of a graph? If so, a graph G having degree sequence S is called a *realization* of S and S is said to be a *graphical* sequence.

The key question was answered independently by Havel and Hakimi.

Theorem: A nonincreasing sequence of nonnegative integers $S : d_1, d_2, \dots, d_n$ is graphical if, and only if, the sequence $S^* : d_2 - 1, d_3 - 1, \dots, d_{d_1+1} - 1, d_{d_1+2}, \dots, d_n$ is graphical.

The proof of this result is a clean “if and only if” argument accessible to most undergraduates. It uses techniques typical of well-known proofs in graph theory like choosing realizations of sequences in a maximal way and utilizing proof by contradiction. Also, the formal proof provides a programmable algorithm for testing whether a sequence of nonnegative integers is graphical. (Received August 24, 2000)