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George W. Cobb* (Gcobb@MtHolyoke.Edu), Department of Mathematics and Statistics, Mount Holyoke College, South Hadley, MA 01075. *The Gauss-Markov Theorem: A Luminous "Two-fer"*.

The Gauss-Markov Theorem offers both a sparkling little crystal of a proof and a distinguished history of applications. The proof involves what I consider the most magical feature of abstraction: its ability to reveal how two "obviously different" structures are in fact one and the same. Applications, which date back to astronomical work in the early 1800s, include myriad instances of fitting equations to data, and remain at the center of much scientific modeling today. Consider two methods for using observed data to estimate the unknown constants of a model. Ordinary least squares (OLS) chooses values that minimize the sum of squared differences between the observed and fitted values. An alternative, best linear unbiased estimation (BLUE), starts with the set of all linear combinations of the observed values, restricts to those combinations that are unbiased, and chooses the one that has smallest variance. The theorem says that, under general conditions, the two methods are in fact one and the same: $OLS = BLUE$. The nicest proof I know represents both methods in terms of orthogonal projections onto subspaces, and establishes a duality between key subspaces determined by the two methods. The OLS projection is simply the dual of the BLUE projection. (Received September 11, 2000)