962-Q1-291 **Luise-Charlotte Kappe** (menger@math.binghamton.edu) and **Friedrich Kluempen*** (fred@math.binghamton.edu), Dept. of Mathematical Sciences, SUNY at Binghamton, Binghamton, NY 13902-6000. *A Letter from Euler to Goldbach: Comments on the Factorization of a Polynomial.*

In the age of Euler, the Fundamental Theorem of Algebra stood only as a conjecture. Attempts were made at proving the Theorem, but support of it was not unanimous. Dunham, in *Euler: The Master of Us All* (MAA, 1999), writes that a number of eminent mathematicians of the day were skeptics. He records that in 1742, Nicholas Bernoulli claimed to have found a fourth degree polynomial with real coefficients, $P(x) = x^4 - 4x^3 + 2x^2 + 4x + 4$, which could not be factored into real quadratics. If his claims were correct, the Fundamental Theorem would indeed be wrong. This is where Euler made his contribution. The factorization of Bernoulli's "counterexample" is recorded in a 1742 letter to Christian Goldbach. The factorization, as reported by Dunham, is

$$\left(x^2 - \left(2 + \sqrt{4 + 2\sqrt{7}}\right) x + \left(1 + \sqrt{4 + 2\sqrt{7}} + \sqrt{7}\right)\right) \left(x^2 - \left(2 - \sqrt{4 + 2\sqrt{7}}\right) x + \left(1 - \sqrt{4 + 2\sqrt{7}} + \sqrt{7}\right)\right).$$

Euler's result can be shown to be correct. However, Euler does not include any supporting work in his correspondence, nor does he leave any hints as to *how* he might have split the quartic into the real quadratics. In *Euler and the Fundamental Theorem of Algebra* (The College Mathematics Journal, **22**, 282-293), Dunham records that Euler published an attempted proof of the Fundamental Theorem of Algebra in 1749. In a section of this proof, Euler establishes the existence of factorization of any real quartic into two real quadratics. Using lines of reasoning as presented in the proof, our talk will reconstruct the process Euler may have used to arrive at the factorization of $P(x)$ above. In addition, using results from Galois Theory, as they can be found in *An Elementary Test for the Galois Group of a Quartic Polynomial* (L.C. Kappe and B. Warren, American Mathematical Monthly, **96**, 133-137), a method will be provided by which the factorization of an irreducible quartic over the rationals into two quadratics over the reals can be directly read off from the coefficients of the quartic. (Received September 07, 2000)