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Dances between continuous and discrete: Euler's summation formula.

Euler developed the Euler-Maclaurin summation formula in the 1730's, using it to make a dazzlingly accurate numerical estimate for the sum of all the reciprocal squares; at that point he was probably convincing himself well beyond reasonable doubt that the sum was exactly $\pi^2/6$, spurring his first proof of this fact.

Euler's summation formula provides breathtakingly effortless acceleration of approximations for many partial and infinite sums of slowly converging or diverging series, despite the formula itself usually diverging. He connected it to Bernoulli numbers, proving all Bernoulli's conjectured sums of powers formulas. He also obtained highly accurate approximations to harmonic partial sums, the gamma constant, sums of logarithms, large factorials (Stirling's series), binomial coefficients, and π . Euler was a wizard at making these connections and calculations.

We will illustrate how Euler's delightfully readable original sources can be used in translation in an undergraduate class unit on "The Bridge between the Continuous and the Discrete", including exercises using powerful mathematical computing software, and interesting further mathematical questions arising from Euler's writings on these topics.

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