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1003-05-1453      **Kevin M Woods\*** ([kwoods@math.berkeley.edu](mailto:kwoods@math.berkeley.edu)), 1606 Prince St, Berkeley, CA 94703.

*Presburger arithmetic, rational generating functions, and the two stamp Frobenius problem.*

As an illustrative example, we examine the following question, and we solve it using rational generating functions: “Given two denominations of stamps,  $a$  cents and  $b$  cents, where  $a$  and  $b$  are relatively prime, what is the largest postal rate that we cannot pay exactly?” This is called the Frobenius problem with two generators. In general, we are interested in sets which can be defined in Presburger arithmetic, that is, sets of natural numbers which can be defined using addition, multiplication by scalars (but not multiplication of variables), Boolean operations (and, or, not), quantifiers ( $\forall, \exists$ ), and comparisons ( $<, =$ ). For example, in our stamp problem with  $a$  and  $b$  given, the set of rates which cannot be paid exactly is the  $x$  such that  $\neg(\exists y, \exists z : x = ay + bz)$  (with  $x, y$ , and  $z$  natural numbers). One can show that sets in Presburger arithmetic are exactly the sets which can be encoded as rational generating functions. In fact, rational generating functions are often an efficient tool to quickly answer questions about these sets (Are they nonempty? What is their cardinality? What is their minimal element?). We survey several results along those lines. (Received October 05, 2004)