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1003-05-472 Nick Cavenagh and Abdollah Khodkar* (akhodkar@westga.edu), Department of Mathematics, State University of West Georgia, Carrollton, GA 30117, and Saad El-Zanati and Charles Vanden Eynden. On a generalization of the Oberwolfach Problem.

One of the best-known problems on factorizations into cycles is the Oberwolfach problem, which was first formulated by Ringel and concerns possible seating arrangements as discussed at a graph theory conference in Oberwolfach, Germany in 1967. The question is if it is possible to seat an odd number m of people at t round tables at which there are a_1, a_2, \ldots, a_t seats (with $a_1 + a_2 + \ldots + a_t = m$ and $a_i \ge 3$) on (m-1)/2 days so that each person sits next to every other person exactly once. Thus the Oberwolfach problem asks for a 2-factorization of the complete graph K_m in which each 2-factor consists of cycles of lengths a_1, a_2, \ldots, a_t .

In this talk, we study the case $m = p^n$, where p is an odd prime. Clearly, if K_{p^n} has a C_t -factor, then t must also be a power of p, say $t = p^k$ with $1 \le k \le n$. In particular, we show that if e_1, e_2, \ldots, e_n is a sequence of nonnegative integers such that the first non-zero term is not one and $\sum_{i=1}^n e_i = (p^n - 1)/2$, then the complete graph K_{p^n} can be decomposed into $e_1 \ C_{p^n}$ -factors, $e_2 \ C_{p^{n-1}}$ -factors, \ldots , and $e_n \ C_p$ -factors. (Received September 15, 2004)