Meeting: 1003, Atlanta, Georgia, SS 26A, AMS-SIAM Special Session on Dynamic Equations on Time Scales; Integer Sequences and Rational Maps, I

1003-11-643 Jeffrey C Lagarias* (lagarias@umich.edu), Department of Mathematics, University of Michigan, Ann Arbor, MI 48109-1109, and N. J. A. Sloane (njas@research.att.com), Information Sciences Research, AT\&T Labs-Research, 180 Park Avenue, Building 103, Florham Park, NJ 07932-0971. Approximate Squaring.
This talk describes results on the approximate squaring map, which sends a real number $x$ to $x($ ceiling $(x))$. If the initial data $x_{0}$ is a rational number, then all iterates are rational. In this case one can raise the question: Must some iterate of $x_{0}$ be an integer? This appears to be the case when $\left|x_{0}\right|>1$, although it seems hard to prove. We show that this happens when the denominator of $x_{0}$ is 2 , and give partial results for larger denominators. We then consider a similar question for the approximate multiplication map, which sends $x$ to $r(\operatorname{ceiling}(x))$, where $r$ is a fixed real number, with $|r|>1$. If $r$ is a rational number, and $x_{0}$ is arbitrary, then one can ask if some iterate is necessarily an integer. We show will occur if the denominator of $r$ is 2 , with one exceptional value of $x_{0}$. Similarly, if the denominator of $r$ is at least 3 , this still holds for almost all input values $x_{0}$. The talk will conclude with some remarks on the case of non-rational inputs to the approximate squaring map. (Received September 25, 2004)

