Meeting: 1003, Atlanta, Georgia, SS 26A, AMS-SIAM Special Session on Dynamic Equations on Time Scales; Integer Sequences and Rational Maps, I

 1003-11-643 Jeffrey C Lagarias* (lagarias@umich.edu), Department of Mathematics, University of Michigan, Ann Arbor, MI 48109-1109, and N. J. A. Sloane (njas@research.att.com), Information Sciences Research, AT&T Labs-Research, 180 Park Avenue, Building 103, Florham Park, NJ 07932-0971. Approximate Squaring.

This talk describes results on the approximate squaring map, which sends a real number x to x(ceiling(x)). If the initial data x_0 is a rational number, then all iterates are rational. In this case one can raise the question: Must some iterate of x_0 be an integer? This appears to be the case when $|x_0| > 1$, although it seems hard to prove. We show that this happens when the denominator of x_0 is 2, and give partial results for larger denominators. We then consider a similar question for the approximate multiplication map, which sends x to r(ceiling(x)), where r is a fixed real number, with |r| > 1. If r is a rational number, and x_0 is arbitrary, then one can ask if some iterate is necessarily an integer. We show will occur if the denominator of r is 2, with one exceptional value of x_0 . Similarly, if the denominator of r is at least 3, this still holds for almost all input values x_0 . The talk will conclude with some remarks on the case of non-rational inputs to the approximate squaring map. (Received September 25, 2004)