Meeting: 1003, Atlanta, Georgia, SS 20A, AMS Special Session on Commutative Algebra, I

1003-13-1047

Daniel L. Katz, dlk@math.ku.edu, and **Emanoil Theodorescu***, theodore@math.missouri.edu.

Hilbert Polynomials Associated to Tor and Ext for Powers of Ideals, Degree and Leading Coefficient. Preliminary report.

Let (R, m, k) be a Noetherian, quasi-unmixed local ring of dimension d. Let I be an ideal and M, N be finite R-modules. It is known that the lengths (if finite) of (i) $\mathrm{Tor_i(M, N/I^nN)}$, (ii) $\mathrm{Ext^i(M, N/I^nN)}$ and (with some restrictions) (iii) $\mathrm{Ext^i(N/I^nN, M)}$ are given by polynomials, if $n >> 0$. In some cases, we get their degrees and leading coefficients. The former are bounded above by, resp., $\max\{\dim \mathrm{Tor_i(M, N)}, \ell_N(I) - 1\}$, $\max\{\dim \mathrm{Ext^i(M, N)}, \ell_N(I) - 1\}$, and a slightly more complicated formula for case (iii). We investigate whether these bounds are the actual degree. In cases (i), (ii) we take $M = k$, $N = R$ and assume I is normal. If $\ell(I) = d$, we show that the Betti and Bass numbers of I^n are polynomials of degree $d - 1$. Thus, analytic spread, rather than height of I, is involved. We give an upper bound for the leading coefficient and, if I is m-primary (normal), we identify it. If $\ell(I) < d$, we get a simpler form for their degree and leading coefficient. In case (i), we get a simple proof of a result by V. Kodiyalam, with $M = k$, $N = R$ and $I = m^t$. In case (iii), we take $M = N = R$, $i = d$, and I m-primary. Then, $\mathrm{length(Ext^d(R/I^n, R))}$ has the same leading term as $\mathrm{length(R/I^n)}$. (Received October 03, 2004)