Meeting: 1003, Atlanta, Georgia, COLLOQ1, AMS Colloquium Lectures:Lecture I

1003-14-5 **Robert Lazarsfeld***, University of Michigan, Department of Mathematics, 2072 East Hall, 525 E Univ Avenue, Ann Arbor MI 48109-1109. *How polynomials vanish: singularities, integrals, and ideals, Part I: How many times does a polynomial vanish at a point?*

Given a polynomial $f(z) \in \mathbf{C}[z]$ in one variable that vanishes at the origin, one learns as an undergraduate how to count the number of times $z = 0$ occurs as a root. For polynomials

$$f = f(z_1, \ldots, z_n) \in \mathbf{C}[z_1, \ldots, z_n]$$

in several variables, the question becomes much more interesting. Now the zeroes of f define a hypersurface in $\mathbf{C}^n$. The naive generalization of the one variable case leads to the notion of the multiplicity of such a hypersurface at a point, whose geometric meaning we will discuss.

However the multiplicity fails to distinguish between polynomials such as

$$z^2 - w^2 \text{ and } z^2 - w^3.$$

A more subtle invariant involves studying for which real numbers $c > 0$ the function

$$\frac{1}{\mid f(z_1, \ldots, z_n) \mid^{2c}}$$

is locally integrable. The critical value of c turns out to govern a surprising number of geometric, analytic, and arithmetic properties of f. (Received March 22, 2004)