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Ted J Suffridge* (ted@ms.uky.edu), Department of Mathematics, University of Kentucky, Lexington, KY 40506. Some extremal properties of holomorphic mappings of the ball onto convex domains. Preliminary report.
Let $B$ denote the unit ball in $\mathbb{C}^{n}$ (the Euclidean ball), $n \geq 2$, and assume $f: B \rightarrow \mathbb{C}^{n}$ is holomorphic, $f(B)$ is convex and $f$ is normalized so that $f(0)=0$ and $D f(0)=I$, the identity. The mapping $f(z)=z /\left(1-z_{1}\right)$ is extremal in the sense that equality is achieved in the inequality $\|z\| /(1+\|z\|) \leq\|f(z)\| \leq\|z\| /(1-\|z\|)$. In addition, if $\varphi(z)$ is the holomorphic automorphism $z->\left(\left(z_{1}+x\right) /\left(1+x z_{1}\right), \sqrt{1-x^{2}} \hat{z} /\left(1+x z_{1}\right)\right)$ where $\hat{z} \in \mathbb{C}^{n-1}$ denotes the last n-1 coordinates of $z$. Then for $f$ as defined above, the Koebe $\operatorname{transform} \Lambda_{\varphi} f(z)$ (i.e. compose $f$ with $\varphi$ and renormalize) has the property $\Lambda_{\varphi} f(z)=f(z)$. It is surprising that this mapping is not an extreme point within the family of normalized holomorphic mappings of the ball onto convex domains. We will discuss the property $\Lambda_{\varphi} g(z)=g(z)$ and the nature of some known extreme points of the family. Some of this work is joint work with John Pfaltzgraff and some is joint with Jerry Muir. (Received September 14, 2004)

