Meeting: 1003, Atlanta, Georgia, SS 22A, AMS Special Session on Spaces of Vector-Valued Functions, I

1003-46-108Ioana Ghenciu*, Mathematics Department, University of Wisconsin, River Falls, River Falls, WI54022, and Paul Lewis, Mathematics Department, University of North Texas, Denton, TX76203-1430. Tensor Products and Dunford-Pettis Sets.

A bounded subset M of the Banach space X is said to be a Dunford-Pettis (DP) subset of X if T(M) is relatively compact in Y whenever $T: X \to Y$ is weakly compact, and M is said to be a strong (or hereditary) DP set if U is a DP subset of the closed linear span [U] of U for each non-empty subset U of M. Note that the unit ball of any infinite dimensional separable reflexive space is a DP subset of C[0, 1] and is not a strong DP set.

Theorem. The Banach space X does not contain a copy of c_0 if and only if every strong DP subset of X is relatively compact.

As a corollary of this theorem, we give an elementary and self-contained proof of a generalization of J. Elton's trichotomy. **Corollary** If X is an infinite dimensional Banach space, then c_0 embeds in X, ℓ_1 embeds in X, or X contains a weakly null Schauder basis (y_n) so that $\{y_n : n \in \mathbb{N}\}$ is not a DP subset of $[y_n : n \in \mathbb{N}]$ and thus $[y_n : n \in \mathbb{N}]$ does not have the DPP. (Received August 07, 2004)