1014-11-244 **Florian Luca**, Instituto de Matemáticas, Universidad Nacional Autonoma de México, Morelia, Michoacan, Mexico, and **Carl Pomerance***, Mathematics Department, Dartmouth College, Hanover, NH 03755. *Irreducible radical extensions and Euler-function chains.* Preliminary report.

Suppose that $K \subset L$ are fields in $\mathbf{C}$ and we can reach L from K by a finite sequence of radical extensions. We are interested here in a complex field M containing L such that M/K is solvable, and we can reach M from K by a sequence of prime-degree radical Galois extensions. It follows from the Kummer theory (Lenstra, unpublished) that such a field exists, and in fact the smallest such field M is also the smallest field M which contains L and which contains the p-th roots of 1 for each prime p dividing $[M : K]$. For example, say $L = \mathbf{Q}(\zeta_{11})$ and $K = \mathbf{Q}$. Then $M = \mathbf{Q}(\zeta_{55})$. In general, if $L = \mathbf{Q}(\zeta_n)$, $K = \mathbf{Q}$, then $M = \mathbf{Q}(\zeta_m)$, where m is the least multiple of n that is divisible by each odd prime dividing $\phi(m)$. Thus, $m = m(n)$ is n times the product of the distinct odd primes that divide $\phi(n)\phi(\phi(n))\dots$ that do not divide n. We show that for each fixed number k, this $m = m(n)$ exceeds n^k for all n in a set of asymptotic density 1. (Received August 30, 2005)