1014-13-671   **Alberto Facchini**, **Wolfgang Hassler**, **Lee Klingler** and **Roger Wiegand***
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68588-0130. *Direct-sum relations among modules over a one-dimensional local ring.*

Let $(R, \mathfrak{m}, k)$ be a one-dimensional reduced Noetherian local ring. Our goal is to describe all direct-sum relations among finitely generated R-modules. In order to do this, we determine the monoid $V(R)$ of isomorphism classes of finitely generated R-modules. If R is a discrete valuation ring, the elementary divisor theorem tells us that $V(R)$ is the free monoid generated by the isomorphism classes of the modules $R/\mathfrak{m}^n, 1 \leq n \leq \infty$. More generally, if R is *Dedekind-like* (roughly speaking, an (A_1)-singularity), then $V(R)$ has a reasonably simple structure which we describe. In the general case, when R is not Dedekind-like, we show that the cardinality of k and the *splitting number* $|\mathrm{Spec}(\widehat{R})| - |\mathrm{Spec}(R)|$ form a complete set of invariants for the monoid $V(R)$.

The problem of finding invariants for the submonoid of $V(R)$ consisting of isomorphism classes of *torsion-free R-modules* appears to be more difficult. We will discuss this problem briefly and give an example illustrating the difficulty. (Received September 22, 2005)