James M. Mailhot* (jim.mailhot@stanfordalumni.org). On Selmer groups in a family of elliptic curves with reducible 2- and 3-torsion and 3-ranks of class groups of quadratic number fields. Preliminary report.
For elliptic curves with reducible $p$-torsion (where $p$ is a prime) we obtain bounds for the $p$-rank of the Selmer group by studying local behavior of the curve at $p, \infty$ and primes of bad reduction. The method of producing the bounds works well in analyzing families of quadratic twists. The flavor of these bounds is different for $p=2$ and $p$ odd. We consider the family of quadratic twists by $d$ of the curve $y^{2}+x y+y=x^{3}+x+2$ (30a1 in Cremona's tables), which has a rational point of order 6, hence reducible 2- and 3-torsion. By choosing $d$ appropriately, we can force the 2 -rank of the Selmer group to be large. In turn, this forces either the 2-rank of the Tate-Shafarevich group or the 3-rank of the class group of $\mathbf{Q}(\sqrt{\mathbf{d}})$ to be large. (Received September 26, 2006)

