Carla D Martin* (carlam@math.jmu.edu), Department of Mathematics and Statistics, MSC
1911, James Madison University, Harrisonburg, VA 22801, and Charles F. Van Loan. The Rank of a Tensor. Preliminary report.
Determining the rank of a matrix is straight-forward using the Singular Value Decomposition (SVD); the number of non-zero singular values in the decomposition equals the rank of a matrix. As computing power increases, many more problems in engineering and data analysis involve computation with tensors, or multi-way data arrays. Most applications involve computing a decomposition of a tensor into a linear combination of rank-1 tensors. Ideally, the decomposition involves a minimal number of terms, i.e., computation of the rank of the tensor. The rank of a general tensor is quite complicated. In fact, computing the rank of an arbitrary tensor is an open problem. In this talk, we provide insight into the connection between tensor rank and eigenvalues of certain matrices.

We begin by illustrating some major differences between matrix rank and tensor rank. Our main contribution is an explicit algorithm to compute the rank of a small subclass of tensors. For clarity, we begin with $2 \times 2 \times 2$ tensors and extend it to $n \times n \times 2$ tensors. These results provide insight into the complexity of tensor rank. Perturbation results will be presented and we conclude with some open problems related to tensor rank. (Received August 25, 2006)

