Bala Krishnamoorthy* (bkrishna@math.wsu.edu), P.O. Box 643113, WSU, Pullman, WA 99164-3113, and William Webb and Nathan Moyer. A Knapsack Cryptosystem Secure Against Attacks using Basis Reduction and Integer Programming.
A knapsack cryptosystem encodes a message $x$ (a $0-1 n$-vector) as $M=a^{T} x$, where $a$ are the knapsack coefficients (public). Its security depends on the fact that $0-1$ knapsack problem is NP-complete. The coefficients of the MerkleHellman system are created from a set $s$ of superincreasing weights $\left(s_{i}>\sum_{j<i} s_{j}\right)$ disguised by a modular multiplication ( $a_{i}=p s_{i} \bmod q ; p, q$ are private). Attacks were proposed on this cryptosystem using Diophantine approximation (Shamir), basis reduction (Lagarias and Odlyzko, and Coster et al.), and integer programming techniques; the superincreasing structure, and low density $\left(n / \log \left(\max _{i} a_{i}\right)\right)$ being the weak points. We propose a knapsack cryptosystem without an underlying superincreasing sequence, and with additional cardinality constraints on $x_{j}$ 's. With $n=r m$, we want one $x_{j}$ from each of $r$ subsets (of size $m$ ) be equal to 1 (in addition to the knapsack equation). For appropriate parameters $(r, m)$, the density of this knapsack is arbitrarily large. Attacks using basis reduction only find near-short vectors in the lattice with increasing probability (and not the shortest vector). Further, standard as well as basis reduction-based integer programming methods fail to solve these instances. (Received September 27, 2006)

