Meeting: 1002, Pittsburgh, Pennsylvania, SS 12A, Special Session on Geometric Analysis and Partial Differential Equations in Subelliptic Structures

1002-43-215 Jeremy T. Tyson* (tyson@math.uiuc.edu), Department of Mathematics, University of Illinois, 1409 West Green St., Urbana, IL 61801. Sharp weighted Young's inequalities and Moser-Trudinger inequalities in groups of Heisenberg type. Preliminary report.

Balogh, Manfredi and Tyson (2003) found the sharp constant A(G) in the conformally invariant Moser-Trudinger inequality on Carnot groups G of homogeneous dimension Q, as follows: the inequality

$$\sup_{\Omega,f} \frac{1}{|\Omega|} \int_{\Omega} \exp(A|f|^{Q/(Q-1)}) < \infty$$

holds with A = A(G), where the supremum is taken over all domains $\Omega \subset G$ with finite Haar measure and all elements f of the horizontal Sobolev space $HW^{1,Q}(\Omega)$ with

$$\int_{\Omega} |\nabla_G f|^Q \le 1,$$

while for each A > A(G) the supremum is infinite. In this talk, we discuss sharp constants for weighted Moser-Trudinger inequalities on groups of Heisenberg type, where Haar measure is replaced by certain measures which are absolutely continuous with respect to Haar measure, with weights which are radial homogeneous with respect to the first-layer coordinates of G. The proofs use sharp Young's inequalities for corresponding weighted convolution operators. As applications, we prove some sharp weighted Moser-Trudinger inequalities for x-symmetric functions on Grushin spaces. (Received September 14, 2004)