Meeting: 998, Houston, Texas, SS 3A, Special Session on Harmonic and Functional Analysis

998-42-218 **Ataollah Askari Hemmat** (asat_h@yahoo,com), Dep. of Math., Vali-Asr University Rafsanjan, Kerman, Iran, and **Jean-Pierre Gabardo*** (gabardo@mcmaster.ca), Dep. of Math. & Stat., McMaster University, Hamilton, Ontario L8S 4K1, Canada. *The uniqueness of shift-generated duals for frames in shift-invariant subspaces.* Preliminary report.

Given an invertible $n \times n$ matrix B and Φ a finite or countable subset of $L^2(\mathbb{R}^n)$, we consider the collection $X = \{\phi(.-Bk) : \phi \in \Phi,\ k \in \mathbb{Z}^n\}$ generating the closed subspace $\mathcal{M}$ of $L^2(\mathbb{R}^n)$. If that collection forms a frame for $\mathcal{M}$, one can introduce two different types of shift-generated dual frames for X, called type I and type II duals respectively. The main distinction between them is that a dual of type I is required to be contained in the space $\mathcal{M}$ generated by the original frame while, for a type II dual, one imposes that the range of the frame transform associated with the dual be contained in the range of the frame transform associated with the original frame. We characterize the uniqueness of both types of duals using the Gramian and dual Gramian operators which were introduced in a paper by Ron and Shen and are known to play an important role in the theory of shift-invariant spaces . We also use these characterizations to show that, under some mild conditions, a dual of type I must necessarily be a Riesz basis for $\mathcal{M}$ while a dual of type II must be fundamental, i.e. it must generates the whole space $L^2(\mathbb{R}^n)$. (Received February 27, 2004)