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(abbarros@mat.ufc.br), Departamento de Matematica, Universidade Federal do Ceara, Campus do Pici, 60455-760 Fortaleza, Ceara, Brazil, and Aldir Brasil (aldir@mat.ufc.br), Departamento de Matematica, Universidade Federal do Ceara, Campus do Pici, 60455-760 Fortaleza, Ceara, Brazil. On the first stability eigenvalue of constant mean curvature hypersurfaces in spheres.

A classical result by Simons (1968) establishes that the first stability eigenvalue λ_1 of a compact minimal hypersurface M^n immersed into an Euclidean sphere \mathbb{S}^{n+1} satisfies that either $\lambda_1 = -n$ (and M is totally geodesic), or $\lambda_1 \leq -2n$. More recently, Wu (1993) and Perdomo (2002) have obtained (independently and using different arguments) that $\lambda_1 = -2n$ if and only if M is a minimal Clifford torus in \mathbb{S}^{n+1} , providing with a very nice characterization of the Clifford torus by the first eigenvalue of the stability operator. Our objective here is to extend these results to the case of constant mean curvature hypersurfaces in \mathbb{S}^{n+1} . Specifically, we will prove that if M has constant mean curvature H, then either $\lambda_1 = -n(1 + H^2)$ (and M is totally umbilic), or

$$\lambda_1 \le -2n(1+H^2) + \frac{n(n-2)}{\sqrt{n(n-1)}} |H| \max |\phi|,$$

where ϕ is the total umbilicity tensor of M. Moreover, equality holds if and only if either n = 2 and M is an H(r)-torus $\mathbb{S}^1(r) \times \mathbb{S}^1(\sqrt{1-r^2})$, or $n \ge 3$ and M is an H(r)-torus $\mathbb{S}^{n-1}(r) \times \mathbb{S}^1(\sqrt{1-r^2})$, with $r^2 \le (n-1)/n$. (Received December 29, 2003)