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**Bruno Nachtergaele** (bxn@math.ucdavis.edu), Department of Mathematics, University of California, Davis, Davis, CA 95616-8633, **Wolfgang Spitzer\*** (spitzer@pims.math.ca), Department of Mathematics, University of British Columbia, Vancouver, BC V6T 1Z2, Canada, and **Shannon Starr** (sstarr@math.mcgill.ca), Department of Mathematics and Statistics, McGill University, 805 Sherbrooke Street West, Montreal, QC H3A 2K6, Canada. *Ferromagnetic ordering of energy levels.*

We study a natural conjecture regarding ferromagnetic ordering of energy levels in the Heisenberg model which complements the Lieb-Mattis Theorem of 1962 for antiferromagnets: for ferromagnetic Heisenberg models the lowest energies in each subspace of fixed total spin are strictly ordered according to the total spin, with the lowest, i.e., the ground state, belonging to the maximal total spin subspace. Our main result is a proof of this conjecture for the spin-1/2 Heisenberg XXX and XXZ ferromagnets in one dimension. Our proof has two main ingredients. The first is an extension of a result of Koma and Nachtergaele which shows that monotonicity as a function of the total spin follows from the monotonicity of the ground state energy in each total spin subspace as a function of the length of the chain. For the second part of the proof we use the Temperley-Lieb algebra to calculate, in a suitable basis, the matrix elements of the Hamiltonian restricted to each subspace of the highest weight vectors with a given total spin. We then show that the positivity properties of these matrix elements imply the necessary monotonicity in the volume. Our method also shows that the first excited state of the XXX ferromagnet on any finite tree has one less than maximal total spin. (Received February 12, 2004)