

Meeting: 999, Nashville, Tennessee, SS 6A, Special Session on Local and Homological Algebra

999-13-210 **Sandra M. Spiroff*** (spiroff@math.utah.edu), University of Utah, Department of Mathematics, Salt Lake City, UT 84112, and **Florian Enescu** and **Catalin Ciuperca**. *Growth of Ideals*.

Let J and I be a pair of ideals in a Noetherian ring such that $J^p \subset I^q$. Obviously $J^{p'} \subset I^{q'}$, for $q' \leq q$ and $p' \geq p$, but what can be said about the sup (p/q) ?

In terms of Algebraic Geometry, let I and J be two ideals of $R = \mathbb{C}[X, Y]$. Set $a_{pq} = \{r \in R : r \cdot J^p \subset I^q\}$. Set $C = \{(p, q) : a_{pq} = R \Leftrightarrow J^p \subset I^q\} \subset \mathbb{Z} \times \mathbb{Z}$. Then C is a cone which contains the x -axis. The slope of the other bordering line is $\sup(q/p)$. R. Lazarsfeld, for one, informally posed the following question: Is this slope rational?

In fact, this question has been affirmatively answered. D. Rees, L. Ratliff & E. Rush, and P. Samuel, all contributed to the solution. However, despite this progress, no one has given a method to actually calculate this number. We seek to do this, approaching the problem from the point of view of the infimum of $\frac{p}{q}$. In the case where the polynomial ring is generated by only two variables and I and J are generated by monomials, we provide a definitive method to solve $\inf(\frac{p}{q})$ when I has at most three generators. We give this algorithm and discuss how to generalize it. (Received August 23, 2004)