

1035-00-10

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In general, when one observes a system where many random inputs come together via finitely many quantities (one sums up the behavior of the system via a number for instance), then this quantity tends to become deterministic when the number of inputs gets very large. The randomness is in some way averaged out, and one sees a non-random system on large scale. Sometimes however, and this is in particular the case for physical systems at the precise temperature where a “phase transition” occurs, it happens that the large-scale behavior remains random.

Physicists have told us for more than twenty years that something special is going on for two-dimensional systems that enable us to describe mathematically this phase transition and the corresponding random systems. The notion of conformal invariance lies at the root of Conformal Field Theory that had been able to predict various features of these two-dimensional phenomena.

In recent years, mathematicians have succeeded in understanding and proving these predictions. An important step was the definition in 1999 of the Stochastic Loewner Evolution by Oded Schramm.

The goal of these lectures will be to describe various general ideas on this topic (due to many people including Greg Lawler, Oded Schramm, Scott Sheffield, Stas Smirnov and myself), trying to make use of the two-dimensional screen to show two-dimensional pictures.

In this second lecture, we shall discuss what it means for a “random coloring” of a planar domain to be conformally invariant, and we shall see that this is a very strong constraint: in fact, it almost allows to characterize these colorings. We shall then describe some lattice models for which asymptotic conformal invariance is now proved and some models for which this is a conjecture. (Received May 01, 2007)