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Samuel Gregory Coskey*, Department of Mathematics, Hill Center, 110 Frelinghuysen Rd,
Piscataway, NJ 08854. *On the classification of torsion-free abelian groups up to
quasi-isomorphism.*

Up to isomorphism, the torsion-free abelian groups of rank n are members of the following standard Borel space: $R_n =$ the subgroups of \mathbb{Q}^n which contain a basis for \mathbb{Q}^n . Hjorth and Thomas have shown that the isomorphism equivalence relations on the R_n increase in complexity with n .

Here is the relevant complexity notion, due to H. Friedman. For E, F Borel equivalence relations on standard Borel spaces X, Y , write $E \leq_B F$ iff there exists a Borel function $f : X \rightarrow Y$ such that $x E y \leftrightarrow f(x) F f(y)$.

We compare the isomorphism equivalence relation on R_n with that of quasi-isomorphism. The definition is: $A, B \in R_n$ are quasi-isomorphic iff A is commensurable with an isomorphic copy $B' \in R_n$ of B . The advantage is that unique decomposition holds with respect to quasi-isomorphism, but fails with respect to isomorphism. This leads one to ask whether quasi-isomorphism is truly less complex than isomorphism.

Our result is a “no” answer, namely that the isomorphism and quasi-isomorphism relations on R_n are incomparable with respect to \leq_B . The proof relies on a recent superrigidity theorem, due to A. Ioana, regarding profinite actions of Kazhdan groups. (Received September 20, 2007)