Gregg Musiker* (musiker@math.mit.edu), 77 Massachusetts Ave., Department of Mathematics, MIT, Cambridge, MA 02139. Combinatorics of Elliptic Curves and Chip-Firing Games.
For a given elliptic curve E over a finite field $F_{q}$, we let $N_{k}=\# E\left(F_{q^{k}}\right)$, where $F_{q^{k}}$ is a $k$ th degree extension of the finite field $F_{q}$. Because the Zeta Function for $E$ only depends on $q$ and $N_{1}$, the sequence $\left\{N_{k}\right\}$ only depends on those numbers as well. More specifically, we observe that these bivariate expressions for $N_{k}$ are in fact polynomials with integer coefficients, which alternate in sign with respect to the power of $N_{1}$. This motivated a search for a combinatorial interpretation of these coefficients, and one such interpretation involves spanning trees of a certain family of graphs. In this talk, I will describe this combinatorial interpretation, as well as applications and directions for future research. One of the important features of elliptic curves which makes them the focus of contemporary research is that they admit a group structure. During the remainder of this talk I will describe chip-firing games, how they provide a group structure on the set of spanning trees, and numerous ways that these groups are analogous to those of elliptic curves. This research has been completed as part of my dissertation work at the University of California, San Diego under Adriano Garsia's guidance. (Received August 02, 2007)

