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**László Babai\*** (laci@cs.uchicago.edu), **Nikolay Nikolov** and **László Pyber**. *Expansion and Product Decompositions of Finite Groups: Variations on a Theme of Gowers*.

Let  $G$  be a finite group of order  $n$  and let  $m$  be the minimum dimension of its irreducible representations over the reals. Let  $X, Y, Z \subseteq G$ . In a recent paper, Tim Gowers proved that if  $|X||Y||Z| \geq n^3/m$  then there exist  $x \in X$ ,  $y \in Y$ , and  $z \in Z$  such that  $xy = z$ .

We observe that Gowers' result can be restated as saying that under the same condition,  $XYZ = G$ . We prove a generalization to the decomposition of  $G$  to a product of  $t \geq 3$  subsets.

Several applications to the area of "bounded generation" follow, resulting in considerably simpler proofs and improved bounds. In particular we prove that all finite simple groups of Lie type are products of 5 Sylow subgroups. Applications to the diameter of Cayley graphs also follow; in particular we extend a recent result of Helfgott.

We also prove an expansion result for two-term products: if  $X, Y \subseteq G$  and  $|Y| \geq \alpha n/m$  then  $|XY| > \min\{n/2, (\alpha/3)|X|\}$ .  
– A strong affirmative answer to a recent problem of Venkatesh and Green about the product growth of moderate size subsets in  $SL_2(q)$  follows. (Received September 16, 2007)