Bala Krishnamoorthy* (bkrishna@math.wsu.edu), PO Box 643113 WSU, Department of Mathematics, Pullman, WA 99164-3113, and William Webb and Nathan Moyer. Discrete Optimization Models for the Number Partitioning Problem.
The number partitioning problem (NPP) is to divide a set of positive integers $a_{1}, \ldots, a_{n}$ into two disjoint subsets such that the difference of the subset sums, called the discrepancy $(\Delta)$, is minimized. NPP is NP-complete, has a well-characterized phase transition, and finds applications in VLSI design, multiprocessor scheduling, cryptography etc. When $a_{j}=U[1, R]$ for some integer $R$, it is known that the optimal $\Delta=O\left(\sqrt{n} 2^{-n} R\right)$. The best known polynomial time approximation algorithm was proposed by Karmarkar and Karp (KK), and gives $\Delta_{K K}=O\left(n^{-0.72 \log n} R\right)$. We propose a mixed integer program (MIP) model for solving NPP. We consider a basis reduction-based reformulation of the MIP in order to handle the typically huge $a_{j}$ 's. We also consider direct application of basis reduction (BR) to NPP, similar to BR attacks on $0-1$ knapsack problems, but on a scaled matrix to find near-optimal solutions (for large values of $R, \Delta$ is typically much bigger than 0 ). Finally, we consider various divide-and-conquer strategies, where smaller NPP sub-problems are solved to optimality, and their solutions are combined to obtain near-optimal solutions for the original NPP instance. (Received July 27, 2007)

