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Andrea S Bruder* (Andrea_Bruder@baylor.edu), Andrea Bruder, Baylor University, One Bear Place #97328, Waco, TX 76798-7328. *Jacobi polynomials ($\alpha=\beta=-1$), their Sobolev orthogonality, and self-adjoint operators.*

It is a well known result by Bochner that, for $-\alpha, -\beta, -\alpha - \beta - 1 \notin \mathbb{N}$, the Jacobi polynomials $\left\{P_n^{(\alpha,\beta)}(x)\right\}_{n=0}^{\infty}$ are orthogonal on \mathbb{R} with respect to a bilinear form of the type

$$(f, g)_{\mu} = \int_{\mathbb{R}} f\bar{g}d\mu,$$

for some measure μ . However, for $\alpha = \beta = -1$, from Favard's theorem, the Jacobi polynomials cannot be orthogonal with respect to a bilinear form of this type for any measure. But are they orthogonal with respect to some "natural" inner product? Indeed, they are orthogonal with respect to a Sobolev inner product. We discuss this Sobolev orthogonality and construct a self-adjoint operator in a certain Hilbert-Sobolev space that has the Jacobi polynomials $\left\{P_n^{(-1,-1)}(x)\right\}_{n=0}^{\infty}$ as a complete set of eigenfunctions. To this end, we show that the quest for the right Hilbert-Sobolev space amounts to the study of the left-definite spaces and operators associated with the Jacobi differential expression. (Received September 19, 2007)