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Thomas J. Osler* (osler@rowan.edu), Math Department, Rowan University, Glassboro, NJ 08028. *Vieta like products involving Fibonacci and Lucas numbers.*

The beautiful infinite product formula of radicals

$$\frac{2}{\pi} = \sqrt{\frac{1}{2}} \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2}}} \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2}}}} \cdots$$

due to Vieta in 1592, is one of the oldest noniterative analytical expressions for π . It is the purpose of this note to prove the following two Vieta-like products

$$\frac{\sqrt{5}F_N}{2N \log \phi} = \sqrt{\frac{1}{2} + \frac{L_N}{4}} \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2} + \frac{L_N}{4}}} \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2} + \frac{L_N}{4}}}} \cdots$$

for N even, and

$$\frac{L_N}{2N \log \phi} = \sqrt{\frac{1}{2} + \frac{\sqrt{5}F_N}{4}} \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2} + \frac{\sqrt{5}F_N}{4}}} \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2} + \frac{\sqrt{5}F_N}{4}}}} \cdots$$

for N odd. Here N is a positive integer, F_N and L_N are the Fibonacci and Lucas numbers, and $\phi = \frac{1+\sqrt{5}}{2}$ is the golden section. (Received September 12, 2007)