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In linear algebra and equally in differential equations, second differences replace second derivatives. This is a terrifically valuable step in our three most basic courses:

Calculus: All students learn how  $dy/dx$  comes from  $\Delta y/\Delta x$ . Until they see a little more, they miss a crucial point. We can choose forward or backward or centered differences (and we do). For second differences the good choice has coefficients 1,  $-2$ , 1 and a division by  $\Delta x$  squared. Why?? This opens up a new understanding of  $y''$ .

Linear Algebra: My favorite matrices have  $-1, 2, -1$  on the three center diagonals. I write them on the day I meet the class, and ask for their properties. (Symmetry is always the first answer.) Are they invertible: yes. What are their pivots, and determinants, and eigenvalues and eigenvectors? All computations are beautiful, and these matrices are everywhere in applications—by their connection to  $-y''$ .

Differential Equations: Solving  $y'' + y = 0$  is a pleasure. The solution is  $y = \cos(t)$  if  $y(0) = 1$  and  $y'(0) = 0$ . What happens when  $y''$  is replaced by a second difference in scientific computing? Again we have choices: Forward Euler, Backward Euler, Leapfrog, Trapezoidal Rule. Those four choices are controlled by their eigenvalues: Spiral out for  $|\lambda| > 1$ , Spiral in for  $|\lambda| < 1$ , Leap onto an ellipse or stay on a circle for  $|\lambda| = 1$ .

That choice is the reality of computational science. I will try to show why. (Received September 21, 2007)