Joseph H Silverman* (jhs@math.brown.edu), Mathematics Department - Box 1917, Brown University, 151 Thayer Street, Providence, RI 02912. On the greatest common divisor of $a^{n}-1$ and $b^{n}-1$. Preliminary report.
A conjecture of Rudnick and Ailon asserts that for multiplicatively independent integers $a>1$ and $b>1$, there are infinitely many exponents $n \geq 1$ such that $\operatorname{gcd}\left(a^{n}-1, b^{n}-1\right)=\operatorname{gcd}(a-1, b-1)$. We present experimental evidence and a heuristic argument for the statement that the number of primes $p<X$ such that $\operatorname{gcd}\left(a^{p}-1, b^{p}-1\right)=\operatorname{gcd}(a-1, b-1)$ is equal to $\pi(X)(1+O(1 / \log X))$. We will also discuss generalized versions of the Rudnick-Ailon conjecture for elliptic curves and other algebraic groups. (Received August 27, 2008)

