Luis A. Caffarelli* (caffarel@math.utexas.edu). Non linear problems involving integral diffusions.

Nonlinear equations with a diffusion component are central to many areas of mathematics and science: fluid dynamics, continuum mechanics, probability, geometry. They quantify the idea that the variable under consideration, U, a temperature, the speed of a flow, a deformation, tries to revert to an "average of itself" in a surrounding infinitesimal neighborhood, resulting in an infinitesimal relation, for instance the heat equation. This averaging effect is what gives elliptic and parabolic equations their stability and regularization properties. In recent years, a lot of interest has surfaced in studying non linear problems where the unknown, U, has global, or at least "non infinitesimal" information, and thus reverts instead to an integral average, giving rise to a non local or integral diffusion equation. Two typical areas where this is encountered is in problems involving surface discontinuities in continuum mechanics (ocean atmosphere interaction, semi permeable membranes, planar crack propagation) and in stochastic processes with jumps (Levy processes), as well as in turbulent flow, material sciences, population dynamics. In this lecture, I plan to describe the type of problems encountered, the difficulties they pose, and give a flavor of the mathematics involved (Received September 07, 2008)