Steven E. Broad* (sbroad@nd.edu), 255 Hurley Hall, Notre Dame, IN 46556. An index formula related to a conjecture of Loewner.

We prove a geometric index formula which produces a "defect" term for the Loewner conjecture about the index of vector fields with isolated zeros of the form $\partial_{\bar{z}}^n f$ for functions $f: \mathbb{C} \to \mathbb{R}$. A recent result of F. Xavier allows the index of such vector fields to be computed in terms of the set of eigenvalues of the Hessian of f in the case n=2. Our result extends this formula to all $n \geq 2$. The Loewner conjecture has a deep connection to the Carathéodory conjecture which states that a smooth, convex embedding of the 2-sphere into \mathbb{R}^3 has at least two umbilies. (Received September 12, 2008)