1046-J1-1965 Douglas Dunham* (ddunham@d.umn.edu), Department of Computer Science, 320 HH, 1114 Kirby Drive, Duluth, MN 55812-3036. The symmetry of M.C. Escher's Circle Limit IV pattern and related patterns. Preliminary report.
M.C. Escher created four patterns in the Poincaré model of hyperbolic geometry. The last one, Circle Limit IV, is an alternating pattern of angels and devils. Escher previously drew a Euclidean pattern of angles and devils as Notebook Drawing 45 and carved a maple ball with that pattern on its surface. The symmetry group of Notebook Drawing 45 is generated by a reflection across the body axis of an angel or devil and a 4 -fold rotation about one of its wing tips. This symmetry group is denoted $\left[4^{+}, 4\right]$ in H.S.M. Coxeter's notation, and $4^{*} 2$ in orbifold notation. On the maple ball some of the angels are indented relative to their surrounding devils, and vice versa. If this indentation is ignored, the symmetry group is $\left[3^{+}, 4\right]$ or $3^{*} 2$, otherwise it is just the dihedral group $D_{2}$, with two perpendicular reflection planes through the poles. In Circle Limit IV, some angels and some devils are only outlined. Close inspection reveals the symmetry of the pattern to be just the dihedral group $D_{3}$, with three reflection lines through the center. If all the outlines were filled in, symmetry group would be $\left[4^{+}, 6\right]$ or $4^{*} 3$. By filling in some outlines, and unfilling others, we can obtain other hyperbolic patterns with more interesting symmetry. (Received September 16, 2008)

