Meeting: 1005, Newark, Delaware, SS 9A, Special Session on Arithmetic Groups and Related Topics

1005-22-76Herbert Abels* (abels@mathematik.uni-bielefeld.de), Universitaet Bielefeld, Fakultaet fuer
Mathematik, Postfach 100131, D-33501 Bielefeld, Germany. A question of Siegel

Let $\Gamma = SL(n, \mathbb{Z})$ and $G = SL(n, \mathbb{R})$. The main result of reduction theory describes a fundamental set S for Γ in G, i.e. a subset S of G such that the natural map $G \to \Gamma \setminus G$ restricted to S, let us call it $\pi: S \to \Gamma \setminus G$, is surjective and has finite fibers. The set S described in reduction theory is called a Siegel set. Siegel asked in 1959 if π has the additional property of preserving the natural metric up to a constant. In recent work with G.A. Margulis we gave a positive answer to this question. Our solution has three interesting features:

1. It works for all reductive groups G over local fields and their corresponding arithmetic subgroups Γ . Positive answers to Siegel's question had been known before for reductive groups over the reals and their arithmetic subgroups (Ding, Ji, Leuzinger).

2.It works for all norm-like metrics, not only for the metrics coming from the symmetric spaces (as considered by the authors above) or from the Bruhat-Tits building, but also for word metrics.

3. The proof gives additional information in reduction theory. (Received January 28, 2005)