Meeting: 1004, Bowling Green, Kentucky, SS 1A, Special Session on Numerical Analysis, Approximation, and Computational Complexity: Interdisciplinary Aspects

Minerva Catral, Department of Mathematics, University of Connecticut, Storrs, CT 06269,
Michael Neumann, Department of Mathematics, University of Connecticut, Storrs, CT 06269,
and Jianhong Xu* (jxu@uwf.edu), Department of Mathematics and Statistics, University of
West Florida, 11000 University Parkway, Pensacola, FL 32514. Matrix analysis of the small-world properties of a ring network.

A recent paper by Higham [4] shows that the small-world properties of a ring network can be analyzed via the matrix perturbation theory together with a finite difference approximation scheme under continuum assumption. This approach, however, leads naturally to asymptotic results for the limiting case when N, the size of the network, is sufficiently large. It is also subject to an additional restriction that ϵ , the parameter interpolating between the completely local and completely global configurations of the network, is of the form $\epsilon = K/N^{\alpha}$, where K > 0 and $\alpha > 1$. Besides, the small-world properties of the network are investigated only for the case when $\alpha = 3$.

Motivated by Highams work, we show in this paper that the small-world properties of the network can be analyzed more rigorously via a purely matrix-theoretic approach, which yields exact results for all N and for all ϵ . These results allow us to further explore the small-world properties of the network. For the more general case when $\epsilon = K/N^{\alpha}$ with $\alpha \neq 3$ and small to moderate N, numerical experiments on our results clearly point to the existence of the small-world phenomenon with the resulting cutoff diagram well conforming to that in [6]. (Received November 16, 2004)